

## Applied Mathematics and Statistics

The Department of Applied Mathematics and Statistics ([www.ams.jhu.edu](http://www.ams.jhu.edu)) is devoted to the study and development of mathematical disciplines especially oriented to the complex problems of modern society. A broad undergraduate and graduate curriculum emphasizes several branches of applied mathematics: *Probability*, the mathematical representation and modeling of uncertainty; *Statistics*, the analysis and interpretation of data; *Operations Research*, the design, analysis, and improvement of actual operations and processes; *Optimization*, the determination of best or optimal decisions; *Discrete Mathematics*, the study of finite structures, arrangements, and relations; and *Scientific Computation*, which includes all aspects of numerical computing in support of the sciences.

*Probability and Statistics* is treated in the curriculum as a single general area, dealing in a unified way with theory and methodology for probabilistic representation of chance phenomena, applications of stochastic modeling to physical and social sciences, formulation of statistical models, fitting of statistical models to data, and interpretation of data. *Operations Research and Optimization* represents a second general area, dealing in unified fashion with the application of optimization theory, mathematical programming, computer modeling, stochastic modeling, and game theory to planning and policy problems such as scheduling, allocation of resources, and facility location. *Discrete Mathematics* includes the traditional themes of graph theory and combinatorics, as well as newer topics arising from modern technological and theoretical developments. The fourth general area, *Computational and Applied Mathematics*, covers topics pertaining to computing, numerical analysis, advanced matrix analysis, and mathematical modeling.

In its fundamental role of representing applied mathematics at Johns Hopkins University, the Department of Applied Mathematics and Statistics is complemented by the Department of Mathematics, with its differing emphasis. Located in the School of Engineering, the Department of Applied Mathematics and Statistics fulfills a special integrative role, stemming in part from the affinity of engineers for applied mathematics and in part from the increasing need for interaction between science and engineering. The mathematical sciences, especially the mathematics of modeling, provide a common language and tools through which engineers can develop closer alliances and cooperation with other scientists.

The department's degree programs include foundational and introductory course work drawing from all areas of the curriculum, along with specialized course work in areas such as probability, statistics, operations research, and optimization. Students, in consultation with their advisers, may develop challenging individual programs. The department emphasizes mathematical reasoning, mathematical modeling, abstraction from the particular, and innovative application, all in a problem-oriented setting. The aim is to prepare graduates for professional careers in the mathematical sciences and related areas, in academic institutions as well as in governmental, industrial, and research organizations.

The undergraduate major in Applied Mathematics and Statistics leads to the B.A. and B.S. degrees. The graduate program leads to the M.A., M.S.E., and Ph.D. degrees. In addition, under a combined bachelor's/master's program, exceptionally able undergraduates may be admitted early to simultaneous graduate work.

### The Faculty

**Gregory L. Eyink**, Professor: mathematical physics, fluid mechanics, turbulence, dynamical systems, partial differential equations, nonequilibrium statistical physics, geophysics and climate.

**James A. Fill**, Professor: probability, stochastic processes, random structures and algorithms.

**Donniell E. Fishkind**, Associate Research Professor: combinatorics, graph theory, matrix analysis.

**Donald Geman**, Professor: image analysis, statistical learning, bioinformatics.

**Alan J. Goldman**, Professor Emeritus: operations research, game theory, optimization, graph theory, facility location.

**Shih-Ping Han**, Professor: optimization, numerical analysis, operations research.

**Paul J. Maiste**, Senior Lecturer: statistics, data mining, statistical genetics.

**Daniel Q. Naiman**, Professor (Chair): statistics, computational probability, bioinformatics.

**Carey E. Priebe**, Professor: statistics, image analysis, pattern recognition.

**Edward R. Scheinerman**, Professor: discrete mathematics, partially ordered sets, random methods, graph theory.

**Fred Torcaso**, Lecturer: stochastic processes, asymptotics and partial differential equations.

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**John C. Wierman**, Professor and Director of the Center for Leadership Education: probability, statistics, random graphs, stochastic processes.

**Laurent Younes**, Professor: Mathematical imaging, shape theory and applied differential geometry, computational probability, statistics.

**Joint, Part-Time, and Visiting Appointments**

**Shiyi Chen**, Professor (Mechanical Engineering): turbulence, computational fluid dynamics, lattice Boltzmann applications, molecular dynamics, flow in porous media.

**Gregory Chirikjian**, Professor (Mechanical Engineering).

**John Goutsias**, Professor (Electrical and Computer Engineering).

**Leslie A. Hall**, Adjunct Associate Professor: combinatorial optimization, integer programming.

**Benjamin F. Hobbs**, Professor (Geography and Environmental Engineering): energy and environmental systems and economics.

**Pablo Iglesias**, Professor (Electrical and Computer Engineering).

**Takeru Igusa**, Professor (Civil Engineering).

**S. Rao Kosaraju**, Edward J. Schaefer Professor (Computer Science): design of algorithms, parallel computation, pattern matching, robotics computational geometry.

**David Marchette**, Lecturer (Naval Surface Warfare Center).

**Michael I. Miller**, Professor (Biomedical Engineering).

**Jerry L. Prince**, Professor (Electrical and Computer Engineering): multi-dimensional signal processing, medical imaging, computational geometry.

**Charles ReVelle**, Professor (Geography and Environmental Engineering): environmental and urban systems, water resources.

**James C. Spall**, Principal Professional Staff (Applied Physics Laboratory, System Identification and Modeling Group).

**James T. Stadter**, Principal Professional Staff (Applied Physics Laboratory, Engineering).

**Mei-Cheng Wang**, Professor (Biostatistics): longitudinal data analysis, survival analysis.

**Facilities**

The department is located in Whitehead Hall. Office space and liberal access to computing facilities are provided to resident graduate students. A Reading/Commons Room provides the opportunity for informal discussions among faculty and graduate students. The University's Milton S. Eisen-

hower Library maintains an excellent collection of literature in the mathematical sciences, including all of the important current journals.

**Undergraduate Programs**

The undergraduate major in Applied Mathematics and Statistics may serve as preparation for employment as an applied mathematician, for graduate study in applied mathematics or related areas, or as a general quantitative training for a career in business, medicine, or other fields. An undergraduate major in Applied Mathematics and Statistics takes an individually tailored program of courses within the department and in the Department of Mathematics (calculus, and perhaps further courses such as differential equations, analysis, complex variables, topology, and modern algebra) and electives in science and engineering. By suitable choice of electives, heavy concentration in a specific field of engineering is possible.

In order to develop a sound program suited to individual needs and interests, the student should consult regularly with the faculty adviser. Sample programs for various options and areas of concentration, and supplemental information, may be obtained from the department office.

With the advice and consent of the faculty adviser, each student constructs an individualized program meeting the requirements below. A written copy of the program should be on file with the faculty adviser, with whom it can be revised and updated from time to time.

**Bachelor's Degrees**

Departmental majors can earn either the B.A. or the B.S. degree by meeting the general requirements of the School of Arts and Sciences or of the School of Engineering, respectively, the general university distribution requirements, and the departmental requirements. (See General Requirements for Departmental Majors, page 46, and Writing Requirement, page 42.)

All courses used to meet the following departmental requirements must be passed with grade of C- or higher:

- 1. Calculus I, II, and III: The courses 110.106-107, 110.108-109, or 110.111-112 can be used to satisfy the Calculus I and II requirements. The courses 110.202 or 110.211-212 satisfy the Calculus III requirements. Advanced placement is acceptable as well.
- 2. A course in linear algebra: 550.291, 110.201, or 110.211-212 is acceptable.
- 3. A course in computing emphasizing numerical/scientific computing: 500.200, 530.106, 550.365,

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or 550.386 is acceptable. (Other courses may be substituted with adviser's approval.)

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4. A course in discrete mathematics: 550.171, 550.371, 550.471, or 550.472 is acceptable.
  5. At least five approved 3 or 4 credit Applied Mathematics and Statistics courses numbered 300 or higher, including an optimization course, typically 550.361, and two courses in probability and statistics, chosen from 550.310, 550.311, 550.420, and 550.430. (Either 550.310 or 550.311 [but not both] can be used for this purpose.) Any course used to satisfy the requirements 1-4 above can be used to meet this requirement. More advanced courses may be substituted with adviser's approval. One course in real analysis (110.405 or higher), abstract algebra (110.401), or differential equations (110.302) may be used toward the total of five courses. Students may choose to write a senior thesis, but the thesis (550.501) does not count toward the five-course total.
  6. Courses coded (Q) totaling 40 credits, of which at least 18 credits must be in courses numbered 300 or higher. (Courses used to meet the requirements above may be counted toward this total.)
  7. For the B.S. degree, at least 15 credits coded (N). Laboratory courses that accompany (N) courses may be used in reaching this total. (Courses used to meet the requirements above may be counted toward this total.)
  8. A sequence of three approved courses in an area of application (outside the department). At least one of these courses must be quantitatively oriented and be at the 300-level or above. Appropriate fields include, but are not restricted to, biology, biomedical engineering, chemistry, civil engineering, computer science, earth and planetary sciences, economics, electrical engineering, mechanical engineering, physics, psychology, sociology, and systems analysis for public decision making.

Requirements 1-8 together constitute a minimal core program, allowing maximum flexibility in planning degree programs. Students often are able to complete a second major during a four-year program or to proceed to the department's combined bachelor's/master's degree program.

It is highly recommended that students establish a concentration (see below) or at least take additional departmental courses, in order to establish a broad foundation for a career as an applied mathematician. Of particular importance are additional courses in optimization (550.362), stochastic processes (550.426), statistics (550.413, 550.432, 550.433, 550.434), numerical methods (550.382),

dynamical systems (550.391), mathematical modeling and consulting (550.400), scientific computing (550.385, 550.386), and investment science (550.442). Students planning to continue to graduate school in an applied mathematics program are encouraged to consider taking one or more graduate-level courses in probability (550.620, 550.621), statistics (550.630, 550.631), optimization (550.661, 550.662), combinatorics (550.671), graph theory (550.672), numerical analysis (550.681), or matrix analysis (550.692).

### Concentrations

The department has established the following optional concentration programs.

#### Probability. Students will take

- 550.420 (550.310/311 may *not* be substituted),
- 550.426, and
- one additional course in probability or statistics at the 400-level (or higher) or real analysis 110.405 (or higher).

#### Statistics. Students will take

- 550.430 (550.310/311 may *not* be substituted), and
- two of the following courses: 550.413, 550.432 through 550.438.

#### Optimization. Students will take

- 550.361, and
- two of 550.362, 550.453, and 550.463.

#### Discrete Mathematics. Students will take

- either 550.471 or 550.472, and
- one additional course from 550.371, 550.471, 550.472, and 550.463.

#### Scientific Computing. Students will take

- two of 550.385, 550.386, and 550.433.

### Capstone Experience

Students may elect to complete a capstone experience. This consists of taking 550.400 Modeling & Consulting in the fall of their senior year followed by a senior thesis (550.501) during the spring. An oral presentation based on the thesis is required.

### Honors

To earn departmental honors, undergraduate majors must earn a GPA of 3.5 or higher in their Applied Mathematics and Statistics courses and do one of the following:

$$2 \times 3 + 4 = 10$$

- complete one of the concentrations described above,
- complete a capstone experience as described above, or
- complete the department's combined bachelor's/master's program.

#### Minor in Applied Mathematics and Statistics

The minor in Applied Mathematics and Statistics should be attractive to students majoring in a variety of disciplines, in both the School of Engineering and the School of Arts and Sciences. The minor provides formal recognition of the depth and strength of a student's quantitative knowledge beyond the minimal requirements of his/her major.

The requirements of the minor in Applied Mathematics and Statistics are the following:

- Completion of an approved program of study containing at least 18 credits in courses coded (Q). The first two courses in calculus (110.106-107 or 110.108-109 or their equivalents) may not be used to fulfill this requirement.
- Among the courses comprising the 18 credits, there must be (a) at least four courses in the Department of Applied Mathematics and Statistics (each of these must be a 3- or 4-credit course); (b) at least three 3- or 4-credit courses at the 300-level or above, of which at least two must be in the Department of Applied Mathematics and Statistics; and (c) an approved semester course based on a high-level computer language (e.g., C, FORTRAN, Pascal, or Java in the courses 600.107, 600.109, or 500.200), or one course which requires one of these courses as a prerequisite.
- The grade in each course counted in fulfillment of requirements for the minor must be at least a C.
- A student wishing to complete a minor in Applied Mathematics and Statistics may obtain more information from the Applied Mathematics and Statistics Department office.

#### The W. P. Carey Minor in Entrepreneurship and Management

Offered by the Center for Leadership Education, the minor in entrepreneurship and management focuses on business and management from a multidisciplinary viewpoint, with a quantitative emphasis. The program offers students a diversified learning experience that emphasizes the concepts, practices, and skills necessary for effective leadership as managers and entrepreneurs in the public and private sectors.

Course descriptions of entrepreneurship and management courses offered by the Center for Leadership Education are provided on page 515.

#### Graduate Programs

A wide variety of advanced courses, seminars, and research opportunities is available in the Department of Applied Mathematics and Statistics. In addition to graduate programs in probability, statistics, operations research, optimization, discrete mathematics and scientific computation, advanced study is possible in interdisciplinary topics in cooperation with other departments, particularly the Departments of Biostatistics, Computer Science, Economics, Geography and Environmental Engineering, Health Services Administration, Mathematics, and Sociology. A graduate student in the Department of Applied Mathematics and Statistics may thus develop a program that suits his/her individual interests and objectives.

Various elements of the graduate program are summarized below. Further information is available from the department office.

#### Admission

To be admitted to an advanced degree program in the department, an applicant must show that he/she has the basic intellectual capacity and has acquired the skills necessary to complete the program successfully within a reasonable period of time. A faculty committee evaluates each applicant's credentials; there are no rigid requirements.

Prospective applicants should submit transcripts of previous academic work, letters of recommendation from persons qualified to evaluate the applicant's academic performance and potential for graduate study, a letter describing anticipated professional goals, and Graduate Record Examination (GRE) scores. Foreign students must submit scores from the Test of English as a Foreign Language (TOEFL). Foreign students applying for teaching assistantships are encouraged to submit scores from the Test of Spoken English (TSE).

Most applicants have undergraduate majors in quantitative fields such as mathematics, statistics, engineering, or a field in the physical sciences, but any major is permitted. Regardless of the major, completion of a program in undergraduate mathematics at least through advanced calculus and linear algebra is essential to begin the normal graduate program.

#### Requirements for the Master's Degree

Students may work toward either the master of arts (M.A.) degree or the master of science in engineering (M.S.E.) degree. Both degrees ordinarily require a minimum of two consecutive semesters of registration as a full-time resident graduate student.

To obtain departmental certification for the master's degree, the student must:

- Complete satisfactorily at least eight one-semester courses of graduate work in a coherent program approved by the faculty adviser. All 600- and 700-level courses are satisfactory for this requirement. Most 400-level courses are also acceptable. All courses must be completed with grades of B- or higher.
- Meet one of the following two options: (a) submit an acceptable research report based on an approved project; or (b) complete satisfactorily two additional one-semester graduate courses, as approved by the faculty adviser.
- Demonstrate a working knowledge of the utilization of computers in the mathematical sciences.

In consultation with the faculty adviser, a candidate for the master's degree plans a complete program of proposed course work and submits it in writing for departmental approval. This should be done early in the first semester of residence.

Doctoral students in other departments may undertake concurrently a master's program in Applied Mathematics and Statistics. Application forms and information are available in the department office.

#### **Requirements for the Bachelor's/Master's Program**

Highly motivated and exceptionally well-qualified undergraduates may apply for admission to the combined bachelor's/master's program in Applied Mathematics and Statistics. Interested students should apply not later than fall semester of their junior year.

The requirements for this program consist of those for the bachelor's and master's programs, as well as:

- At least two consecutive semesters of full-time residence after admission to the program.
- Satisfactory completion of at least 145 course credits.

As part of the application for admission to this program, a student submits a current transcript and a complete proposed program of course work which will meet the requirements. Application forms are available online at <http://gradadmin.as.jhu.edu/graduateapplication/default.cfm>.

#### **Requirements for the Ph.D. Degree**

The objective of the department's Ph.D. program is to produce graduates who are broadly educated in applied mathematics and statistics and who can work at the current frontiers of their chosen special-

ized disciplines. The introductory phase of graduate study acquaints the student with a spectrum of topics, provides an opportunity to fill gaps in his or her background, and affords a close view of the doctoral research process and of potential research areas and advisers. Continuation to advanced study and dissertation research is based upon favorable evaluation of preparedness and potential. A formal evaluation is normally made during the January intersession in the student's second year of graduate study. A further evaluation is made in the third year. The culmination of the program is the doctoral dissertation, representing an original and significant contribution to knowledge in applied mathematics.

In addition to fulfilling the University requirement of a minimum of two consecutive semesters of registration as a full-time resident graduate student, the student must accomplish the following to obtain departmental certification for the Ph.D.:

- Pass the Introductory Examination, normally offered immediately before each semester.
- Pass the Ph.D. Candidacy Examination. This oral examination is normally taken in the third year of residency. The scope of the exam will be governed by a syllabus prepared by the student with the help of the student's mentor or adviser.
- Pass the Graduate Board Oral Examination, normally taken in the third year of residence.
- Complete satisfactorily a one-year elective course (or the equivalent) in some area of application of Applied Mathematics and Statistics.
- Acquire teaching experience under the supervision of the faculty.
- Demonstrate a working knowledge of the utilization of computers in Applied Mathematics and Statistics.
- Complete a program of original research and its clear exposition in a written dissertation. The dissertation must be approved by at least two faculty readers and be certified by them to be a significant contribution to knowledge and worthy of publication in scholarly journals. The candidate defends the dissertation in a public examination held under the auspices of the Department.

Additional details on these items may be found at the department's Web site.

#### **Course Program**

The most common way for students to gain the knowledge and skills to succeed in the Ph.D. program is through coursework. In consultation with his or her adviser, each student will develop a program of proposed coursework. The relevant

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courses for the Ph.D. are of three types: basic graduate-level courses, additional specialized courses appropriate to the student's field of research, and an elective one-year course selected to broaden the student in applied mathematics. To promote a well-rounded education and record, all full-time graduate students are expected to enroll in an appropriate number of courses for their stage in the program. Students are required to enroll in and attend 550.600, the Applied Mathematics and Statistics Department Seminar, every semester. Grades of B- or better (or equivalent level of performance in pass/fail courses) are expected of all department graduate students in their coursework.

**Basic Courses:** All students are encouraged to master basic material in:

- probability (550.620), statistics (550.630), and stochastic processes (550.426);
- optimization (550.661, 550.662);
- numerical and matrix analysis (550.681, 550.692); and
- discrete mathematics (550.671, 550.672).

Normally, a student will have completed at least eight basic courses by the end of the fourth semester of residence.

**Specialized Courses:** Each student takes advanced courses appropriate to the proposed area of dissertation research. Sample programs in the areas of probability/statistics, operations research/optimization, discrete mathematics, scientific computation, and numerical analysis are given online at the department's Web site, but a student with different goals is free to propose an appropriate program meeting the approval of the research adviser.

**Elective Courses:** A one-year graduate course (or the equivalent) in a field distinct from the student's

specialized area is required. This is a minimal requirement. Students are encouraged to take more than two semesters of elective coursework, either covering one area in depth or covering two areas. Typical areas in other departments are biology, econometrics, mathematical economics, mathematical ecology, computational geometry, systems theory, health systems, mathematics, facility location, psychometrics, and physics. These courses may complement or supplement the student's previous experience, but if a student has no previous experience in an area some elementary coursework may be necessary as a prerequisite to acceptable graduate level courses. Although students are strongly encouraged to take the elective courses outside the department, with the approval of the adviser they may be chosen within the department, provided they are 600- or 700-level courses in a field clearly distinct from the student's specialized area.

#### Financial Assistance

A limited number of teaching and research assistantships providing full tuition and a competitive academic year stipend are available to qualified full-time Ph.D. candidates. Furthermore, the following special fellowships are awarded:

- The Rufus P. Isaacs Fellowship, named in honor of a late member of the faculty acclaimed for his contributions to operations research.
- The Abel Wolman Fellowship, awarded by the Whiting School in honor of a pioneering engineering school faculty member.
- The Charles and Catherine Counselman Fellowship, generously endowed by Hopkins alumnus Charles Counselman.

In addition, summer employment opportunities are often available within the university and in the Baltimore Washington corridor.

## Undergraduate Courses

*Prospective students are invited to discuss with individual instructors the aims and prerequisites of their courses; formal prerequisites are listed to indicate the level and type of background expected and may be waived by the instructor for a student with suitable alternative preparation.*

### 550.100 (E,Q) Introduction to Applied Mathematics and Statistics

A seminar-style series of lectures and assignments to acquaint the student with a range of intellectual and professional activities performed by applied mathematicians and statisticians. Problems arising in applied mathemat-

ics and statistics are presented by department faculty and outside speakers. Prerequisite: one semester of Calculus. Naiman 1 credit spring

### 550.111 (Q,E) Statistical Analysis I

First semester of a general survey of statistical methodology. Topics include descriptive statistics, probability models, random variables, expectation, sampling, the central limit theorem, classical and robust estimation of location, confidence intervals, hypothesis testing, two-sample problems, introductory analysis of variance, introductory nonparametric methods. Three lectures and a conference weekly. Some use of computing with the Minitab

statistical package, but prior computing experience not required. Prerequisite: four years of high school mathematics. Students who may wish to undertake more than two semesters of probability and statistics should consider 550.420-430.

Fishkind, Maiste 4 credits

#### 550.112 (Q,E) Statistical Analysis II

Second semester of a general survey of statistical methodology. Topics include least squares and regression analysis, correlation, further nonparametric methods, chi-square tests, the likelihood concept, decision theory, Bayesian inference, time series, simultaneous equations, sample survey design. Prerequisite: 550.111. Students who may wish to undertake more than two semesters of probability and statistics should consider 550.420-430.

Fishkind, Maiste 4 credits

#### 550.122 (Q) Chance and Risk

The course will help students develop an appreciation of probability and randomness, and an understanding of its applications in real life situations involving chance and risk. Applications, controversies, and paradoxes involving risk in business and economics, health and medicine, law, politics, sports, and gambling will be used to illustrate probabilistic concepts such as independence, conditional probability, expectation, and variance. The course is intended primarily for humanities and social science majors. There is no prerequisite beyond high school mathematics; in fact, the course is not open to students who have taken two semesters of calculus.

Wierman 3 credits spring

#### 550.171 (Q,E) Discrete Mathematics

Introduction to the mathematics of finite systems. Logic; Boolean algebra; induction and recursion; sets, functions, relations, equivalence, and partially ordered sets; elementary combinatorics; modular arithmetic and the Euclidean algorithm; group theory; permutations and symmetry groups; graph theory. Selected applications. The concept of a proof and development of the ability to recognize and construct proofs are part of the course. Prerequisite: four years of high school mathematics.

Fishkind, Scheinerman, Torcaso 4 credits

#### 550.230 (Q,E) Introduction to Biostatistics

A self-contained course covering various data analysis methods used in the life sciences. Topics include types of experimental data, numerical and graphical descriptive statistics, concepts of (and distinctions between) population and sample, basic probability, fitting curves to experimental data (regression analysis), comparing groups in populations (analysis of variance), methods of modeling probability (contingency tables and logistic regression). Prerequisite: 3 years of high school mathematics.

Maiste 4 credits spring

#### 550.251 (Q,E) Mathematical Models for Decision Making

As society's enterprises and technologies grow more and more complex, their operation and planning rely increasingly on mathematics-based analyses. This course is an

introduction to management science and the quantitative approach to decision making. Emphasis on model development and case studies, using spreadsheets or other computer software, applied to a variety of problems in manufacturing, transportation, finance, and general management. Prerequisite: one semester of calculus.

Pang, Torcaso 4 credits spring

#### 550.291 (Q,E) Linear Algebra and Differential Equations

An introduction to the basic concepts of linear algebra, matrix theory, and differential equations that are used widely in modern engineering and science. Intended for engineering and science majors whose program does not permit taking both 110.201 and 110.302. Prerequisites: one year of calculus, computing experience.

Pang, Torcaso 4 credits

#### 550.303 (E, Q) Differential Equations

The aim of this course is to present the formulation, solution, and qualitative understanding of differential equations of various types that are used to model real-world phenomena. Topics include first-order, second-order and higher-order differential equations, series solutions, Laplace transforms, systems of equations, numerical methods, and nonlinear equations. Prerequisite: Calculus II.

Eyink, Torcaso 4 credits spring

#### 550.310 (Q,E) Probability and Statistics for the Physical and Information Sciences and Engineering

An introduction to probability and statistics at the calculus level, intended for engineering and science students planning to take only one course on the topics. Students are encouraged to consider 550.420-430 instead. Combinatorial probability, independence, conditional probability, random variables, expectation and moments, limit theory, estimation, confidence intervals, hypothesis testing, tests of means and variances, goodness-of-fit. Prerequisite: one year of calculus. Recommended corequisite: multivariable calculus. Students cannot receive credit for both 550.310 and 550.311.

Fishkind, Maiste 4 credits

#### 550.311 (Q,E) Probability and Statistics for the Biological and Medical Sciences and Engineering

An introduction to probability and statistics at the calculus level, intended for students in the biological sciences planning to take only one course on the topics. The basic scope of this course is similar to 550.310, with an emphasis on examples and problems in the biological sciences. Students are encouraged to consider 550.420-430 instead. Combinatorial probability, independence, conditional probability, random variables, expectation and moments, limit theory, estimation, confidence intervals, hypothesis testing, tests of means and variances, and goodness-of-fit will be covered. Prerequisite: One year of calculus; Corequisite: 110.202 recommended. Students cannot receive credit for both 550.310 and 550.311.

Fishkind, Geman, Maiste, Younes 4 credits

**550.331 (E,Q) Introduction to Mathematical Finance**

The principal aim of this course is to provide the mathematical ideas leading up to the now famous Black-Scholes formula for options pricing. Topics to be covered will include: basic probability, normal random variables, Brownian motion, interest rates, the arbitrage theorem, pricing of various types of options. Prerequisites: Calculus I, II and III.

Naiman 4 credits

**550.361-362 (Q,E) Introduction to Optimization**

An introductory survey of optimization methods, supporting mathematical theory and concepts, and application to problems of planning, design, prediction, estimation, and control in engineering, management, and science. Study of varied optimization techniques including linear programming, network-problem methods, dynamic programming, integer programming, and nonlinear programming. Prerequisites for 550.361: one year of calculus, linear algebra, computing experience. Prerequisites for 550.362: 550.361 and multivariable calculus. Appropriate for undergraduate and graduate students without the mathematical background required for 550.661.

Goldman, Hall 4 credits 361 fall;

362 alternate springs

**550.371 (Q,E) Cryptology and Coding**

A first course in the mathematical theory of secure and reliable electronic communication. Cryptology is the study of secure communication: How can we ensure the privacy of messages? Coding theory studies how to make communication reliable: How can messages be sent over noisy lines? Topics include finite field arithmetic, error-detecting and error-correcting codes, data compressions, ciphers, one-time pads, the Enigma machine, one-way functions, discrete logarithm, primality testing, secret key exchange, public key cryptosystems, digital signatures, and key escrow. Prerequisites: 550.171 (110.204 with permission of instructor), linear algebra, computing experience.

Fishkind, Scheinerman 4 credits spring

**550.382 (Q,E) Introduction to Numerical Methods**

General tools and techniques for the numerical solution of mathematical problems and analysis of solution algorithms, treated at the level of first-year calculus and elementary linear algebra. Topics include floating-point arithmetic, interpolation and approximation, Gaussian elimination for linear equations, numerical differentiation and integration, numerical solution of difference and ordinary differential equations, and boundary value problems. Prerequisites: one year of calculus, linear algebra, computing experience. Corequisites: 110.202 and 550.291 or approved alternative.

Han 4 credits

**550.385 (E,Q) Scientific Computing: Linear Algebra**

A first course on computational linear algebra and applications. Topics include floating-point arithmetic, algorithms and convergence, Gaussian elimination for linear systems, matrix decompositions (LU, Cholesky, QR), iterative methods for systems (Jacobi, Gauss-Seidel), and

approximation of eigenvalues (power method, QR-algorithm). Theoretical topics such as vector spaces, inner products, norms, linear operators, matrix norms, eigenvalues, and canonical forms of matrices (Jordan, Schur) are reviewed as needed. Matlab is used to solve all numerical exercises; no previous experience with computer programming is required. Prerequisites: Calculus III, and 550.291 or approved alternative (e.g., 110.201).

Fishkind 4 credits fall

**550.386 (E, Q) Scientific Computing: Differential Equations**

A first course on computational differential equations and applications. Topics include floating-point arithmetic, algorithms and convergence, root-finding (midpoint, Newton, and secant methods), numerical differentiation and integration, and numerical solution of initial value problems (Runge-Kutta, multistep, extrapolation methods, stability, implicit methods, and stiffness). Theoretical topics such as existence, uniqueness, and stability of solutions to initial-value problems, conversion of higher-order/non-autonomous equations to systems, etc., will be covered as needed. Matlab is used to solve all numerical exercises; no previous experience with computer programming is required. Prerequisites: Calculus III, and 550.291 or approved alternative (e.g., 110.201).

Eyink 4 credits spring

**550.391 (Q,E) Dynamical Systems**

Mathematical concepts and methods for describing and analyzing linear and nonlinear systems that evolve over time. Topics include boundedness, stability of fixed points and attractors, feedback, optimality, Liapounov functions, bifurcation, chaos, and catastrophes. Examples drawn from population growth, economic behavior, physical and engineering systems. The main mathematical tools are linear algebra and basic differential equations. Prerequisites: multivariable calculus, linear algebra, computing experience.

Eyink, Goldman, Scheinerman 4 credits fall

**550.400-401 (Q,E,W) Mathematical Modeling and Consulting**

Formulation, analysis, interpretation, and evaluation of mathematical models. Synthesis of ideas, techniques, and models from mathematical sciences, science, and engineering. Case studies to illustrate basic features of the modeling process. Project-oriented practice and guidance in modeling techniques, research techniques, and written and oral communication of mathematical concepts. Prerequisites: probability, statistics, and optimization at the 300-level or higher.

Maiste, Priebe, Torcaso, Wierman 4 credits

**550.413 (Q,E) Applied Statistics and Data Analysis**

An introduction to basic concepts, techniques, and major computer software packages in applied statistics and data analysis. Topics include numerical descriptive statistics, observations and variables, sampling distributions, statistical inference, linear regression, multiple regression, design of experiments, nonparametric methods, and sample surveys. Real-life data sets are used in lectures and computer



assignments. Intensive use of statistical packages such as S+ to analyze data. Prerequisite: 550.112 or equivalent. Maiste, Naiman 4 credits

**550.420 (Q,E) Introduction to Probability**  
Probability and its applications, at the calculus level. Emphasis on techniques of application rather than on rigorous mathematical demonstration. Probability, combinatorial probability, random variables, distribution functions, important probability distributions, independence, conditional probability, moments, covariance and correlation, limit theorems. Students initiating graduate work in probability or statistics should enroll in 550.620. Prerequisite: one year of calculus. Recommended corequisite: multivariable calculus. Fill, Wierman 4 credits fall

**550.426 (Q,E) Introduction to Stochastic Processes**  
Mathematical theory of stochastic processes. Emphasis on deriving the dependence relations, statistical properties, and sample path behavior including random walks, Markov chains (both discrete and continuous time), Poisson processes, martingales, and Brownian motion. Applications that illuminate the theory. Prerequisite: 550.420. Fill, Wierman 4 credits spring

**550.430 (Q,E) Introduction to Statistics**  
Introduction to the basic principles of statistical reasoning and data analysis. Emphasis on techniques of application. Classical parametric estimation, hypothesis testing, and multiple decision problems; linear models, analysis of variance, and regression; nonparametric and robust procedures; decision-theoretic setting, Bayesian methods. Prerequisite: 550.420. Marchette, Naiman, Priebe 4 credits spring

**550.432 (Q,E) Linear Statistical Models**  
The general linear model in matrix terms. Techniques of applications, with use of statistical computer packages. Multiple regression, polynomial regression, stepwise regression, multicollinearity, reparametrization, normal correlation models and analysis; basic and multifactor analysis of variance, fixed and random effects. Prerequisites: 550.430, 550.291 or approved alternative. Maiste, Naiman, Torcaso 3 credits

**550.433 (Q,E) Monte Carlo Simulation and Reliability**  
Applications of numerical analysis to statistics. Linear least squares; random number generation; Monte Carlo techniques; analysis of variance; time series computations; numerical integration. Emphasis on computational aspects relevant to practical statistical problems. Prerequisites: 550.430, computing experience. Naiman 3 credits

**550.434 (Q,E) Nonparametric and Robust Methods**  
Statistical methodology without strict parametric model assumptions. Exploratory data analysis; linear rank statistics; tests of independence, symmetry, location differences, scale differences, and regression alternatives; chi-square and Kolmogorov-Smirnov goodness-of-fit tests; association analysis; order statistics; nonparametric confi-

dence intervals; nonparametric analysis of variance; influence curves; robust estimation of location and regression parameters. Some use of statistical computer programs. Prerequisite: 550.430. Priebe 3 credits

**550.435 (Q,N) Bioinformatics and Statistical Genetics**  
Biological research has evolved to the point where complex quantitative tools are playing an ever increasing role. The aim of this course is to survey various computational and statistical methodologies that have been put into play in the analysis of biological data to better understand biological phenomena. A large spectrum of biological applications used to motivate the choice of topics. Probabilistic methods, as well as algorithmic ideas related to the assembly, alignment, and matching of DNA sequences, will be developed, and statistical inference methods for making genotype to phenotype connections will be presented. Prerequisites: 550.310, 550.311 or equivalent. Maiste, Naiman 3 credits

**550.436 (E,Q) Data Mining**  
Data mining is a relatively new term used in the academic and business world, often associated with the development and quantitative analysis of very large databases. Its definition covers a wide spectrum of analytic and information technology topics, such as machine learning, artificial intelligence, statistical modeling, and efficient database development. This course will review these broad topics, and cover specific analytic and modeling techniques such as advanced data visualization, decision trees, neural networks, nearest neighbor, clustering, logistic regression, and association rules. Although some of the mathematics underlying these techniques will be discussed, our focus will be on the application of the techniques to real data and the interpretation of results. Because use of the computer is extremely important when "mining" large amounts of data, we will make substantial use of data mining software tools to learn the techniques and analyze datasets. Prerequisite: 550.310 or equivalent. Recommended prerequisite: 550.413. Maiste 4 credits

**550.437 (E,Q) Information, Statistics, and Perception**  
Statistical inference, inductive learning and information theory together provide a cohesive framework for machine perception. Various problems in image analysis and computational biology will be analyzed in this context in both theory and practice (working algorithms). Examples include visual tracking, object recognition, texture modeling, neural decoding and gene expression. Prerequisites: 550.310 or 550.311 as well as some additional exposure to probability and statistics, e.g., 550.420 and/or 550.430. Geman 3 credits

**550.438 (E,Q) Statistical Methods for Computer Intrusion Detection**  
This course will give an introduction to the data and methodologies of computer intrusion detection. The focus will be on statistical and machine learning approaches to detection of attacks on computers. Topics will include

network monitoring and analysis, including techniques for studying the Internet, and estimating the number and severity of attacks; network-based attacks such as probes and denial of service attacks; host-based attacks such as buffer overflows and race conditions; malicious code such as viruses and worms. Statistical pattern recognition methods will be described for the detection and classification of attacks. Techniques for the visualization of network data will be discussed. The book will be supplemented with readings of various articles. Prerequisite: 550.310 or 550.311, or equivalent.

Marchette 3 credits

#### 550.439 (E,Q) Time Series Analysis

Time series analysis from the frequency and time domain approaches. Descriptive techniques; regression analysis; trends, smoothing, prediction; linear systems; serial correlation; stationary processes; spectral analysis. Prerequisites: 550.310, 550.311 or equivalent calculus-based probability course, 110.201 or 550.291 and mathematical maturity.

Torcaso 3 credits

#### 550.442 (E,Q) Investment Science

Intended for upper-level undergraduate and graduate students, this course offers a rigorous treatment of the subject of investment as a scientific discipline. Mathematics is employed as the main tool to convey the principles of investment science and their use to make investment calculations for good decision-making. Topics covered in the course include the basic theory of interest and its application to fixed-income securities, cash flow analysis and capital budgeting, mean-variance portfolio theory, and the associated capital asset pricing model, utility function theory and risk analysis, derivative securities and basic option theory, portfolio evaluation. The student is expected to be comfortable with the use of mathematics as a method of deduction and problem solving. Prerequisites: One year of calculus, an introductory course in probability and statistics (such as 550.310, 550.311 or its equivalent). Some familiarity with optimization is desirable but not necessary.

Staff 4 credits

#### 550.453 (Q,E) Mathematical Game Theory

Mathematical analysis of cooperative and noncooperative games. Theory and solution methods for matrix games (two players, zero-sum payoffs, finite strategy sets), games with a continuum of strategies, N-player games, games in rule-defined form. The roles of information and memory. Selected applications to economic, recreational, and military situations. Prerequisites: multivariable calculus, probability, linear algebra.

Goldman 4 credits alternate springs

#### 550.457 (Q,E) Topics in Operations Research

Study in depth of a special mathematical or computational area of operations research, or a particular application area. Recent topics: decision theory, mathematical finance, optimization software.

Goldman 3 credits

#### 550.463 (Q,E) Network Models in Operations Research

In-depth mathematical study of network flow models in operations research, with emphasis on combinatorial approaches for solving them. Introduction to techniques for constructing efficient algorithms, and to some related data structures, used in solving shortest-path, maximum-volume flow, and minimum-cost flow problems. Emphasis on linear models and flows, with brief discussion of non-linear models and network design. Prerequisites: 550.361 or 550.661.

Fishkind 4 credits

#### 550.471 (Q,E) Combinatorial Analysis

Counting techniques: generating functions, recurrence relations, Polya's theorem. Combinatorial designs: Latin squares, finite geometries, balanced incomplete block designs. Emphasis on problem solving. Prerequisites: linear algebra, one year of calculus.

Fishkind, Scheinerman 4 credits fall

#### 550.472 (Q,E) Graph Theory

Study of systems of "vertices" with some pairs joined by "edges." Theory of adjacency, connectivity, traversability, feedback, and other concepts underlying properties important in engineering and the sciences. Topics include paths, cycles, and trees; routing problems associated with Euler and Hamilton; design of graphs realizing specified incidence conditions and other constraints. Attention directed toward problem solving, algorithms, and applications. One or more topics taken up in greater depth. Prerequisite: linear algebra.

Fishkind, Scheinerman 4 credits spring

#### 550.475 (Q,E) Computational Algebra

Commutative algebra approached from a computational point of view. The theory and application of Groebner bases and related algorithms (e.g., Buchberger's algorithm) for solving many standard problems related to polynomials in several variables. Many elementary and important ideas from algebra will be covered, including elimination theory, implicitization, ideals, Hilbert's Nullstellensatz, and an introduction to elementary algebraic geometry. A key focus of the course is on the algorithmic treatment of classical algebraic problems as well as problems from robotics and automatic theorem proving. Computer algebra packages will be demonstrated and used by the students. Prerequisites: Linear Algebra (550.291 or 110.201), Discrete Math (550.171) or equivalent.

Naiman 4 credits

#### 550.480 (E,Q) Shape and Geometry

This class will review the basic definitions and properties of curves and surfaces, and their relation to the description and characterization of 2D and 3D shapes. Intrinsic local and semi-local descriptors, like the curvature of the second fundamental form will be introduced, with an emphasis on the invariance of these features with respect to rotations, translations, etc. Extension of this point of view to other class of linear transformations will be given, as well as other types of shape descriptors, like moments or medial axes. Prerequisites: Linear Algebra and Calculus III.

Younes 3 credits

**550.491 (Q,E) Applied Analysis for Engineers and Scientists**

This course will cover techniques and applications of differential and integral analysis that are important for advanced work in engineering and science, including partial differential equations and transform methods. Prerequisites: Calculus 1, 2, 3, and either 550.291 and 500.303, or 110.201 and 110.302.

Eyink 4 credits fall

**550.493 (E,Q) Mathematical Image Analysis**

This course introduces a series of mathematical concepts for low level image processing and the numerical algorithms that are derived from them. These include linear and non-linear smoothing and enhancement, PDE-based isotropic and anisotropic filters, variational energy-minimization methods, data analysis and decomposition methods allowing low-level image understanding: standard image transforms (Fourier, cosine, wavelets), techniques of principal and independent component analysis. Prerequisites: elementary calculus (110.108-109 or equivalent), linear algebra (110.201 or equivalent)

Younes 3 credits

**550.500 Undergraduate Research**

Reading, research, or project work for undergraduate students. Pre-arranged individually between students and faculty. Recent topics and activities: percolation models, data analysis, course development assistance, dynamical systems. Offered each semester.

**550.501 Senior Thesis**

Preparation of a substantial thesis based upon independent student research, under the pre-arranged supervision of at least one faculty member in Applied Mathematics and Statistics. Offered each semester.

**550.510 Readings in Actuarial Mathematics**

Independent reading of mathematical topics pertinent to the insurance industry and actuarial profession. Arranged individually between students and faculty. Possible topics: risk theory, financial mathematics, mathematical demography, survival models, forecasting.

Fill, Maiste, Wierman

**Graduate Courses**

**550.600 Applied Mathematics and Statistics Department Seminar**

A variety of topics discussed by speakers from within and outside the university. Required of all resident department graduate students. Offered each semester.

**550.601 Applied Mathematics and Statistics Entering Graduate Student Workshop**

This course is intended for new doctoral students in the Department of Applied Mathematics and Statistics to cover fundamental topics necessary for their graduate education.

Torcaso 3 hours fall

**550.620 Probability Theory I**

Probability as a mathematical discipline, including introductory measure theory. Axiomatic probability, combinatorial probability, random variables, conditional probability, independence, distribution theory, expectation, Lebesgue-Stieltjes integration, variance and moments, probability inequalities, characteristic functions, conditional expectation. Prerequisites: 110.405 and 550.420, or equivalents.

Fill, Wierman 4 hours fall

**550.621 Probability Theory II**

Probability at the level of measure theory, focusing on limit theory. Modes of convergence, Poisson convergence, three-series theorem, strong law of large numbers, continuity theorem, central limit theory, Berry-Esseen theorem, infinitely divisible and stable laws. Prerequisites: 550.620, 110.405, or equivalents.

Fill, Wierman 3 hours spring

**550.626 Stochastic Processes II**

Measure-theoretic treatment of stochastic processes, focusing on continuous parameter processes. Brownian motion and diffusion, renewal processes, continuous-time martingales, stationary processes. Existence and construction, structure, sample path behavior, and asymptotic properties. Prerequisites: 550.426, 550.621.

Fill, Wierman 3 hours

**550.630 Statistical Theory I**

The fundamentals of mathematical statistics. Distribution theory for statistics of normal samples; exponential statistical models; sufficiency principle; least squares, maximum likelihood, and UMVU estimation; hypothesis testing, the Neyman-Pearson lemma, likelihood ratio procedures; the general linear model, the Gauss-Markov theorem, multiple comparisons; contingency tables, chi-square methods, goodness-of-fit; nonparametric and robust methods; decision theory, Bayes and minimax procedures. Prerequisite: 550.420 or 550.620.

Naiman, Priebe, Younes 4 hours fall

**550.631 Statistical Theory II**

Advanced concepts and tools fundamental to research in mathematical statistics and statistical inference: asymptotic theory; optimality; various mathematical foundations. Prerequisite: 550.630.

Naiman, Priebe, Younes 3 hours spring

**550.632 Multivariate Statistical Theory**

Theory of statistics when data are in the form of multivariate observations. The multivariate normal distribution; Wishart distributions; inference on means, Hotelling's T2; multivariate linear models; regression, ANOVA; inference on covariances; classification and discrimination; principal components; canonical correlations; canonical variables. Prerequisites: 550.630, 550.692.

Naiman, Priebe 3 hours

**550.633 Time Series Analysis**

Time series analysis from the frequency and time domain approaches. Descriptive techniques; regression analysis;