MIT 1965

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ROBERT LEON PENDLETON, PH.D.
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EGBERT BRIESKORN, DR.RER.NAT.
BRIAN HARTLEY, PH.D.
ANTHONY WILLIAM KNAPP, PH.D.
RICHARD GUSTAVUS LARSON, PH.D.
EDGAR GEORGE KENNETH LOPEZ-ESCOBAR, PH.D.

Instructors

NORTON STARR, PH.D.
THOMAS JOSEPH LARDNER, PH.D. (*Absent*)
WARREN LEE MAY, PH.D.
JAMES ROBERT GEISER, S.B.
ANGUS CARMICHAEL KERR-LAWSON, PH.D.
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JOHN JAMES UCCI, PH.D.
DIRK VAN DALEN, PH.D.
HARRY PRINCE ALLEN, PH.D.
THOMAS FULCHER BICKEL, PH.D.
THOMAS BLOOM, PH.D.
JOEL MARK COHEN, SC.B.
HARRY DYM, PH.D.
VINCENT WILLIAM GIAMBALVO, S.B.
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DAVID PAUL KRAINES, PH.D.
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ALFRED BERRY MANASTER, PH.D.
STEPHEN KINSLEY PARROTT, PH.D.
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HERBERT LOUIS WILLKE, JR., PH.D.

Professors Emeriti

HENRY BAYARD PHILLIPS, PH.D., LL.D.
Professor of Mathematics, Emeritus
RAYMOND DONALD DOUGLASS, PH.D., SC.D.
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DIRK JAN STRUIK, PH.D.
Professor of Mathematics, Emeritus
SAMUEL DEMITRY ZELDIN, PH.D.
Associate Professor of Mathematics, Emeritus

MATHEMATICS

This Department offers a variety of programs to those interested in mathematics primarily as a pure science or for the additional purpose of applying it to other sciences, to fields of engineering, or to business.

THE UNDERGRADUATE CURRICULUM
Mathematics (Course XVIII)

Each undergraduate's program is arranged through continuous collaboration between the individual student and his Faculty Counselor, subject only to a minimum uniform set of Course requirements. In most cases, the undergraduate curriculum in mathematics is preparatory to further professional training at the graduate level. For this purpose each student is encouraged both to obtain a substantial grounding in each of the fundamental branches of mathematics and also to explore certain fields of application, in order to establish a basis for the selection of an appropriate field of graduate specialization.

Whatever the ultimate objective — preparation for teaching, participation in the research programs of an industrial or government-sponsored organization, or a position as consultant in a business or in a modern high-speed computation center — the immediate educational aims are to provide an understanding of a substantial part of the existing body of mathematical knowledge and an ability to impart this knowledge to others. But most important, the Department hopes to inspire a deep interest in the discovery or invention of new mathematics or in the application of mathematics to a new field.

Students wishing to work with a small group of fourth-year students under the supervision of a faculty member or to do individual work with a member of the Department may elect either to participate in a senior mathematics seminar or to write a thesis. These possibilities furnish excellent opportunities for students to develop initiative. The experience gained from active participation in a seminar conducted by a research mathematician may be particularly valuable for a student planning to pursue graduate work in any branch of mathematics.

The undergraduate Course leads to the degree of Bachelor of Science in Mathematics. The curriculum requirements for the degree are given below.

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CURRICULUM OF MATHEMATICS PROGRAM 1

GENERAL INSTITUTE REQUIREMENTS (*see page 175*) — *Total units*

			Total units
	Specific science and humanities and social science subjects		128
	The Science Distribution Requirement will be satisfied by 18.034, which is required in the Departmental Program, plus appropriate subjects (see the list on page 44) totalling		24
	Laboratory Requirement (see the list on page 45)		12

DEPARTMENTAL PROGRAM

Required Subjects

18.034	DIFFERENTIAL EQUATIONS	3 0 9		
18.21	ANALYSIS	3 0 9		
18.22	ANALYSIS	3 0 9		
	[1]LANGUAGE	4 0 8		48

Restricted Electives

	An integrated program of advanced mathematics subjects at the undergraduate or graduate level, including the following at least		48	
	Either			
18.20T	LINEAR ALGEBRA	3 0 9		
	or the following combination:			
18.25	MODERN ALGEBRA *and*	3 0 9		
18.26	MODERN ALGEBRA	3 0 9		
	Either			
18.24T	ELEMENTARY DIFFERENTIAL GEOMETRY	3 0 9		
	or the following combination:			
18.241	INTRODUCTION TO TOPOLOGY *and*	3 0 9		
18.242	RIEMANNIAN GEOMETRY	3 0 9		
	Senior seminar in mathematics, thesis, or advanced mathematics subject at the undergraduate or graduate level		12	
	Advanced professional subjects in the Schools of Science or Engineering at the undergraduate or graduate level, at least		24	
			at least	84
	Unrestricted Electives[2]		up to	64
	Total units required for the S.B. degree			360

CURRICULUM OF MATHEMATICS PROGRAM 2

GENERAL INSTITUTE REQUIREMENTS (*see page 175*) — *Total units*

			Total units
	Specific science and humanities and social science subjects		128
	The Science Distribution Requirement will be satisfied by 8.03T and 18.034, which are required in the Departmental Program, plus appropriate subjects (see the list on page 44) totalling		12
	Laboratory Requirement (see the list on page 45)		12

DEPARTMENTAL PROGRAM

Required Subjects

8.03T	ELECTRICITY AND MAGNETISM	4 0 8		
18.034	DIFFERENTIAL EQUATIONS	3 0 9		
18.21	ANALYSIS	3 0 9		
18.651	INTRODUCTION TO APPLIED MATHEMATICS	3 0 9		
	[1]LANGUAGE	4 0 8		60

Restricted Electives

	An integrated program of advanced mathematics subjects at the undergraduate or graduate level, including the following, at least		48	
	Either			
18.20T	LINEAR ALGEBRA	3 0 9		
	or the following combination:			
18.25	MODERN ALGEBRA *and*	3 0 9		
18.26	MODERN ALGEBRA	3 0 9		
	Either			
18.22	ANALYSIS *or*	3 0 9		
18.652	INTRODUCTION TO APPLIED MATHEMATICS	3 0 9		
	Either			
18.60T	INTRODUCTION TO FLUID MECHANICS *or*	3 0 9		
18.66T	INTRODUCTION TO ELASTICITY	3 0 9		
	Senior seminar in mathematics, thesis, or advanced mathematics subject at the undergraduate or graduate level		12	
	Advanced professional subjects in the Schools of Science or Engineering at the undergraduate or graduate level, at least		24	
			at least	84
	Unrestricted Electives[2]		up to	64
	Total units required for the S.B. degree			360

[1] The language must be French, German, or Russian. Any student who has taken two or more years of French, German, or Russian before coming to M.I.T. or who demonstrates to his Faculty Counselor his ability to read fluently mathematical literature in one of these languages may substitute 12 units of unspecified elective subjects for the language requirement.

[2] Programs of students in Course XVIII may always be arranged to leave available 12 units of unrestricted elective in the first year.

Transfer from one program to the other may be made at any time. In either program the minimum algebra requirement is the one-semester subject LINEAR ALGEBRA (18.20T), but students wishing more work in algebra as undergraduates should take the two-semester subject sequence in MODERN ALGEBRA (18.25, 18.26). For students following Program 1, the minimum requirement in geometry is the one-semester subject ELEMENTARY DIFFERENTIAL GEOMETRY (18.24T), but students wishing more work in geometry should take instead the two one-semester subjects INTRODUCTION TO TOPOLOGY (18.241) and RIEMANNIAN GEOMETRY (18.242). Students interested in applied mathematics normally would follow Program 2, with INTRODUCTION TO APPLIED MATHEMATICS (18.652) recommended. It is strongly recommended that the requirements in algebra, analysis, and introduction to applied mathematics be completed by the end of the third year, and that Physics 8.04T be included at an early stage in Program 2. The language requirement should be completed as early as possible.

It is recommended that undergraduate students planning to pursue graduate work in elasticity, fluid mechanics, and related fields include the following elective subjects:

INTRODUCTION TO FUNCTIONS OF A COMPLEX VARIABLE (18.27, 18.28)
MECHANICS I (8.711)

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GRADUATE STUDY IN MATHEMATICS

On the graduate level, the Department offers programs leading to the degrees of Master of Science, Doctor of Philosophy, or Doctor of Science in Mathematics.

Students may select their programs from a broad range of subjects, descriptions of which appear elsewhere in this Catalogue. Numerous informal seminars supplement the basic program. There are also a weekly Mathematics Colloquium sponsored by M.I.T., Brandeis, and Harvard and a weekly Colloquium on Applied Mathematics and Mechanics, each of which brings guest lecturers from other institutions.

Candidates whose primary interest is in the field of pure mathematics will ordinarily take most of their subjects in the Mathematics Department. In addition to their advanced specialization, they will be encouraged to acquire breadth by taking basic subjects in analysis, algebra, geometry, and topology.

Candidates whose primary interest is in the field of applied mathematics will be encouraged to study important aspects of one or more engineering or scientific fields closely related to research in applied mathematics, in addition to their studies in mathematics.

Candidates primarily interested in pure mathematics which is basically motivated by applications and developments in theoretical physics or other disciplines will take special programs involving substantial breadth in both pure mathematics and the relevant cognate discipline.

Assistance or even collaboration in problems in pure or applied mathematics which are being investigated by members of the staff may constitute part of a graduate student's program.

Entrance Requirements for Graduate Study

The admission requirements are those set forth in Section 3 of this Catalogue, with the following additions and exceptions:

Science. Students are expected to have one year of college-level natural science.

Mathematics. Students will normally be expected to present an undergraduate mathematics program approximating that required of undergraduate mathematics majors at M.I.T.

The Graduate Degrees

MASTER OF SCIENCE IN MATHEMATICS

For the Master's degree, a student must take not less than 66 units including four 12-unit graduate ("A") subjects offered by the Department as well as submitting an acceptable thesis.

In addition, either as an undergraduate or as a graduate student, each student is expected to have completed four subjects (or their equivalents) from the following group:

METHODS OF APPLIED MATHEMATICS FOR ENGINEERS (18.15)
PROBABILITY (18.18T)
LINEAR ALGEBRA (18.20T)
or
MODERN ALGEBRA (18.26)
ANALYSIS (18.21, 18.22)
ELEMENTARY DIFFERENTIAL GEOMETRY (18.24T)
or
RIEMANNIAN GEOMETRY (18.242)
INTRODUCTION TO TOPOLOGY (18.241)
MODERN ALGEBRA (18.25)
INTRODUCTION TO FUNCTIONS OF A COMPLEX VARIABLE (18.27, 18.28)
INTERMEDIATE DIFFERENTIAL EQUATIONS (18.29)
INTRODUCTION TO FLUID MECHANICS (18.60T)
INTRODUCTION TO APPLIED MATHEMATICS (18.651, 18.652)
INTRODUCTION TO ELASTICITY (18.66T)

Of the four subjects elected from this group, two may be counted for credit towards the Master's degree if taken while the student is a graduate student at M.I.T.

DOCTOR OF PHILOSOPHY AND DOCTOR OF SCIENCE IN MATHEMATICS

The general requirements for the Doctor of Philosophy and Doctor of Science degrees are those set forth in Section 3. As soon as possible after beginning graduate work, each student who plans to take a doctorate in mathematics should elect one of the following two plans of study:

Plan A, Qualifying Examination. A written qualifying examination on topics in algebra (primarily linear algebra), intermediate real analysis (based on a syllabus comparable to the current 18.21–18.22) and complex analysis (topics from 18.31). This examination will be given each year in November and in May. A graduate student is required to take the qualifying examination not later than the beginning of his second year (November). A student who has completed the necessary subjects before becoming a graduate student here should take this examination during his first year. The examination can be waived for students presenting to the Departmental Graduate Committee satisfactory evidence of having learned the material; for example, by having achieved sufficiently high standing in classes in these subjects at M.I.T. or elsewhere.

Plan B, General Examination. A written examination consisting of two parts; the first on Methods of Applied Mathematics (based on 18.653–18.654), and the second on application to

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physical subjects (based on 18.651–18.652, 18.60T–18.61, and 18.66T–18.67). The examination will be held in February, and students should apply at the Graduate Office by December 15 to take it, usually in their second graduate year. This written examination, together with an oral examination to be arranged with a prospective thesis supervisor selected by the student, constitutes the general examination under Plan B. The requirement of a written examination may be waived by petitioning the Departmental Graduate Committee; approval requires that the student's grades and general academic performance show proficiency in the methods of applied mathematics and in their application to specific physical subjects. The prospective thesis supervisor may also require further evidence of the candidate's readiness to perform research work.

It is expected that students planning to do doctoral theses in elasticity, fluid dynamics, and related fields will in most cases elect to follow Plan B.

Students in both Plan A and Plan B will take additional subjects, chosen in consultation with an adviser, from the Department's 12-unit graduate ("A") subjects other than those included in the qualifying examination.

In special circumstances a student may receive permission to work under another plan (Plan N), in which regular Course requirements and written examinations in mathematics subjects are omitted. Students admitted to this program are given oral examinations on their general mathematical progress.

The Minor program may be in any field, including mathematics, in which the Institute offers advanced work. It should be started early in the student's graduate career.

Doctoral candidates are required to demonstrate proficiency in two foreign languages, which are usually chosen from French, German, and Russian. The language requirements should also be fulfilled early in the student's program.

Before the student begins work on his thesis, he is asked to submit to the Departmental Graduate Committee a brief general outline on the proposed thesis, carrying the approval of the faculty member who is his thesis adviser. When the Committee has approved the thesis proposal, the Department names two faculty members to work with the student and his adviser. The thesis is expected to represent original research and to meet the standards ordinarily required for publication in one of the standard journals devoted to research in mathematics or in a closely allied field.

Fellowships and Assistantships

Fellowships and teaching and research assistantships are available to graduate students in mathematics. Detailed information about these opportunities may be obtained from the Department.

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17.621 *The Development of Political Organizations* (A)
Prereq.: 17.722
Year: G (1) 3–0–6
Examination of the factors involved in the origins, development, and maintenance of various types of political organizations. Particular attention given to the creation of political parties and the maintenance of local party organizations in underdeveloped areas. *Weiner*

17.623 *Social Organization and Political Change in Developing Areas* (A)
Prereq.: 17.721 or 17.722
Year: G (1) 3–0–6
Analysis of principles of social organization, social stratification, and social mobility as they affect various aspects of political development such as political participation, political leadership, political organization, legitimacy, and political integration. (*Not offered 1965–66.*) *Weiner*

17.631 *Theories of Political Development* (A)
Prereq.: 17.52 or 17.721
Year: G (1) *Arr.*
Study of various analytical models of the political process in transitional societies. Systematic examination of factors influencing political behavior in changing societies; functions of politics in such societies. *Pye*

17.64 *Political Science Seminar*
Prereq.: 17.223
Year: U (2) 3–0–5
Seminar for students in political science program with members of the faculty to explore various aspects of selected problems in political science. Coordination of thinking from the varying points of view of political science and development of ability to deal with issues of decision-making. (*Not offered 1965–66.*) *Weiner*

17.652 *Field Research in Political Development* (A)
Prereq.: 17.131
Year: G (2) 3–0–6
Seminar in field research methods for political research in developing areas. Intended only for students who will begin field research the following academic year. *Weiner*

17.71 *Reading Seminar in Social Science*
Prereq.: ——
Year: U (1, 2) *Arr.*
Reading and discussion of special topics in the fields of social science. Open to advanced undergraduates by arrangement with individual staff members. *Staff*

17.711 *Reading Seminar in Social Science* (A)
Prereq.: ——
Year: G (1) *Arr.*

17.712 *Reading Seminar in Social Science* (A)
Prereq.: ——
Year: G (2) *Arr.*
Reading and discussion of special topics in the fields of social science. Open to advanced graduate students by arrangement with individual staff members. ——

17.721 *Concept Formation and Research Technique in Political Behavior*
Prereq.: ——
Year: G (1) *Arr.*
Requirements for testable concepts and innovations in research technique relating to attitudes, institutions, decision-making. Case studies on key problems of national, international, and comparative processes. *Staff*

17.722 *Theory of Political Processes*
Prereq.: 17.721
Year: G (2) 3–0–6
Analysis of the theoretical assumptions underlying the political process at different levels and in different systems with a view to identifying structures and functions; representing these elements by suitable concepts or variables and examining their interrelationships in some scheme or model. Testing of different theoretical models by the insights they may yield about the same political practices or institutions as well as by their ability to bring out contrasts among different processes or types of government. *Pool*

17.731 *Introduction to Mathematical Models in the Social Sciences*
Prereq.: 17.721
Year: G (1) 3–0–6
Consideration in detail of three or four models in various areas of social science. Some drilling in the mathematics involved. Specially intended for people with little mathematical background. *Lehrer*

17.732 *Statistics for the Social Sciences*
Prereq.: ——
Year: G (2) 3–0–6
Elementary subject in statistics, with particular emphasis on topics useful in social science. Introductions to descriptive statistics, sampling, estimation, testing of hypotheses, correlation and contingency, and nonparametric methods. No mathematical prerequisites. *Lehrer*

17.75 *Prediction of Social and Political Change*
Prereq.: 14.02 or 17.52 or 21.403 or 21.525
Year: U (1) 3–0–5
Theories of social change. Trends, fluctuations, and unique events. Technological changes and inventions as causes and effects of social change. Prediction of population trends and their consequences. Psychological biases in prediction. Predictions explicit and implicit in policy making. Interaction with an adversary and problems of intelligence prediction. *Iklé*

18.
Mathematics

18.00 *Elementary Number Theory*
Prereq.: ——
Year: 1 (1) 2–0–4
Elementary theory of whole numbers, including such topics as divisibility, prime and composite numbers, greatest common divisors, solutions of equations in integers, the congruence notation and its application, sums of squares. Emphasis on mathematical rigor and methods of proof. *G. B. Thomas*

18.01T *Calculus*
Prereq.: ——
Year: 1 (1) 4–0–8
Study of functions of one variable. Graphical representation, properties of continuous functions. Differen-

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tiation, with applications to curve sketching, maximum-minimum problems. Mean-value theorem. Indefinite and definite integration, fundamental theorem; applications to finding areas, simple volumes, arc length, simple surface areas. Logarithm and exponential functions. Technique of indefinite integration. Approximation methods, Taylor series, applications to calculation of the elementary functions. Thomas, *Calculus and Analytic Geometry, 3d edition.* *Mattuck*

18.02T *Calculus*
Prereq.: 18.01T
Year: 1 (2) 4–0–8
Study of functions of several variables. Vector algebra in three-space. Vector functions of one variable, analysis of space motion; Kepler's laws. Scalar functions of two and three variables; partial differentiation, gradient, exact differentials, line integrals, multiple integrals, areas, volumes, surface areas. Surface integrals, Green's theorem, and Stokes' theorem. Thomas, *Calculus and Analytic Geometry, 3d edition.* *Mattuck*

18.03 *Calculus*
Prereq.: 18.02T
Year: U (1) 3–0–6
Infinite series. Complex numbers. Linear algebra. Probability. Munkres, *Elementary Linear Algebra.* Thomas, *Calculus and Analytic Geometry, 3d edition.* Brunk, *Introduction to Mathematical Statistics.* (*Credit is not permitted in both 18.03 and 18.034.*) (*Not offered after 1965–66.*) *H. Rogers*

18.034 *Differential Equations*
Prereq.: 18.02T
Year: U (2) 3–0–9
Elementary linear algebra, linear differential equations with constant coefficients, geometry of solutions, existence and numerical computation of solutions, non-linear equations, phase plane, linear systems with variable coefficients, series solutions. (*If credit is received in 18.03 or 18.04, then credit is not permitted in 18.034. During the academic year 1965–66, 18.034 is open only to qualified first-year students.*) *L. N. Howard*

18.04 *Differential Equations*
Prereq.: 18.03
Year: U (1, 2) 3–0–6
Treatment of ordinary differential equations. Series solutions. Kaplan, *Ordinary Differential Equations.* (*Credit is not permitted in both 18.04 and 18.034.*) (*Not offered after 1965–1966.*) *H. Rogers*

18.05T *Advanced Calculus for Engineers* (*A, except Courses II, VI, VIII, XIII, XVI, XVIII, XXII*)
Prereq.: 18.034 or 18.04
Year: G (1); U (1, 2, S) 3–0–9
Functions of a complex variable; calculus of residues; conformal mapping. Ordinary differential equations; integration by power series; Bessel and Legendre functions. Expansion in series of orthogonal functions, including Fourier series. Hildebrand, *Advanced Calculus for Applications.* *Hildebrand*

18.06T *Advanced Calculus for Engineers* (*A, except Courses II, XVI, XVIII, XXII*)
Prereq.: 18.05T
Year: G. (1, 2, S) 3–0–9
Vector analysis: orthogonal curvilinear coordinates. Calculus of variations. Solution of classical equations of mathematical physics, including applications of conformal mapping and the Laplace transformation. Partial differential equations; characteristics. Hildebrand, *Advanced Calculus for Applications.* *Hildebrand*

18.07 *Review of Mathematics*
Prereq.: ——
Year: G (S) 8–0–12
Review of calculus and differential equations. Restricted to selected officers of the U. S. Army, Air Force, Navy and Coast Guard.) Douglass and Zeldin, *Calculus and Its Applications;* Kaplan, *Ordinary Differential Equations.* *Staff*

18.10T *Applications of Probability and Random Variables*
Prereq.: 18.02T
Year: U (2) 3–0–9
Theory of probability and random variables on an elementary level with emphasis upon mathematical models having wide applicability. Discrete and continuous random variables, joint distributions, derived distributions, expected value, difference equations, generating functions, some elementary aspects of queuing theory and Markov processes, decision making, and some probabilistic methods in the control of operations. Stress on applications from a wide choice of fields. *Wadsworth*

18.11 *Operations Research* (A)
Prereq.: 18.05T or 18.21; 18.10T
Year: G (1) 3–0–9
Theory of probability and random variables; emphasis on mathematical models having important applications in the field of operations research. Problems involving maximization with limited total effort, search theory, queuing theory, time series, stochastic processes, difference equations, and generating functions. Examples of physical and laboratory operations; illustrations from the fields of production, inventory control, reliability, maintenance, and military operations research. (*Not offered 1965–66.*) *Wadsworth*

18.13 *Numerical Analysis* (A)
Prereq.: 18.05T or 18.651
Year: G (1) 3–2–7

18.14 *Numerical Analysis* (A)
Prereq.: 18.13
Year: G (2) 3–2–7
Introduction to the theory and practice of the solution of equations, interpolation, numerical differentiation and integration, and the numerical solution of ordinary and partial differential equations. Additional topics from the following: summation of series, least-squares methods, smoothing of data, Gaussian quadrature, Chebyshev approximation, harmonic analysis, approximation by exponential functions and by rational functions, determination of characteristic numbers of matrices. Hildebrand, *Introduction to Numerical Analysis.* (*18.14 not offered 1965–66.*) *Hildebrand*

18.15 *Methods of Applied Mathematics for Engineers* (A, except *XVIII*)
Prereq.: 18.05T
Year: G (1, 2) 3–0–9
Operations with matrices and determinants; linear vector spaces; characteristic-value problems. Techniques of calculus of variations; constraints, direct methods. Formulation and treatment of integral equations; Green's function; analytical and numerical methods of solution. Hildebrand, *Methods of Applied Mathematics.* *Hildebrand*

(A) indicates a subject given primarily for graduate students. (R) indicates a subject restricted to special groups because of content. Other notations are described on the first page of this Section.

Grad flag

Do we need?

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18.16J ***Heuristic Programming and Artificial Intelligence* (A)**
(Same subject as 6.544J)
Prereq.: 6.41 or 6.539 or *18.18T* or 18.89; 18.034 or 18.04
Year: G (2) 3–0–9
The problem of making machines behave intelligently. Application of machines to problems of learning, induction, pattern-recognition, game-playing, theorem-proving, neural nets, self-organizing systems, and other areas where problems have been solved by heuristic programming. Problems in administration and allocation of effort within programs. Models of cognitive processes. Machine aids for human problem-solving. *Ability to program a computer required.* *Minsky*

18.17 ***Combinatorial Analysis* (A)**
Prereq.: 18.03 or *18.034*
Year: G (1) 3–0–9
Survey of combinatorial problems of current research interest. Topics include enumerative analysis, theory of partitions, graph theory, knot theory, network flow, theory of matroids, matrices of zeros and ones, combinatorial designs, representations of the symmetric group, combinatorial methods in statistical mechanics, combinatorial geometry, Ramsey's theorem, matching theory, asymptotic enumeration. Varying content from year to year, so that students may take the subject in successive years. (*Not offered 1965–66.*) —

18.18T ***Probability***
Prereq.: 18.03 or 18.034
Year: U (2) 3–0–9
Random variables and expectations. Laws of large numbers, fluctuation theory, recurrent events, Markov chains, birth-and-death processes, introduction to the general theory of stochastic processes. *H. Rogers*

18.182 ***Introduction to Stochastic Processes* (A)**
Prereq.: 18.18T
Year: G (2) 3–0–9
Independence. Zero-one laws of Borel-Cantelli and Kolmogorov. Chebyshev's and Kolmogorov's inequalities. Distribution functions and transforms. Classical limit theorems (deMoivre-Laplace, Poisson, strong law of large numbers). Random walks, Markov chains, Poisson processes, Brownian motion and diffusion. Connection with differential and integral equations. Stationary chains and prediction. Boltzmann's equation. *Ray*

18.20T ***Linear Algebra***
Prereq.: 18.034 or 18.04
Year: U (1, 2) 3–0–9
Vector spaces, euclidean spaces, linear transformations, linear equations, matrices, determinants, quadratic forms, canonical forms of matrices. If 18.20 is taken after credit has been received in 18.25, then credit is *not* permitted in both 18.20 and 18.26. Hoffman and Kunze, *Linear Algebra.* *Curtis*

18.21 ***Analysis***
Prereq.: 18.03 or 18.034
Year: U (1) 3–0–9

18.22 ***Analysis***
Prereq.: 18.20T or *18.26;* 18.21
Year: U (2) 3–0–9
Real numbers and euclidean n-space; open, closed and compact subsets of n-space. Continuous and differentiable functions. Lebesgue integration in euclidean space. The Weierstrass approximation theorem; Fourier series. Jacobians and the inverse function theorem. Change of variables in multiple integrals. Stokes' theorem. *Ambrose*

18.23 ***Senior Seminar in Mathematics***
Prereq.: 18.21
Year: U (1, 2) 3–0–9
Seminars for senior mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussion by students on topics taken from current journals or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite. *Staff*

18.24T ***Elementary Differential Geometry***
Prereq.: 18.22
Year: U (2) 3–0–9
Plane and space curves. First and second differential form of a surface. Theorems of Meusnier and Euler. Lines of curvature, asymptotic lines, conjugate lines, geodesics. Theorems of Gauss and Codazzi. Developable surfaces, surfaces of rotation, Liouville surfaces. Differential parameters. Problems of mapping. *D. W. Anderson*

18.241 ***Introduction to Topology***
Prereq.: 18.20T or *18.25;* 18.22
Year: U (1) 3–0–9
Topological spaces, connectedness, compactness, continuous functions, separation axioms, product space and function space topologies, plus several additional topics to be chosen by the instructor. *Peterson*

18.242 ***Riemannian Geometry***
Prereq.: 18.22, 18.241
Year: U (2) 3–0–9
First and second fundamental forms, geodesics, curvature, spaces of constant curvature, rigidity, Gauss-Bonnet theorem (with emphasis on one- and two-dimensional cases). Riemann surfaces. Algebraic curves. *Curtis*

18.25 ***Modern Algebra***
Prereq.: 18.034 or 18.04
Year: U (1) 3–0–9

18.26 ***Modern Algebra***
Prereq.: 18.25
Year: U (2) 3–0–9
Groups, subgroups, factor groups, homomorphism theorems; rings and ideals, factorization theory for Euclidean rings; vector spaces and elementary field theory. Modules over a ring of operators. Applications to vector spaces and linear algebra. Normal forms for matrices. Quadratic forms. Tensor and Grassman algebras. *Pendleton*

18.27 ***Introduction to Functions of a Complex Variable* (A, *except XVIII*)**
Prereq.: 18.034 or 18.04
Year: G (1) 3–0–9

18.28 ***Introduction to Functions of a Complex Variable* (A, *except XVIII*)**
Prereq.: 18.27 or 18.31
Year: G (2) 3–0–9
Complex numbers, analytic functions, Riemann surfaces for certain functions, Cauchy's theorem, singularities, residues, contour integral, conformal mapping, Schwarz-Christoffel transformation, series and sequences, analytic continuation, harmonic functions, conjugate functions, the gamma function, second order linear differential equations and special functions, Laplace transforms, asymptotic series, saddlepoint method, Hilbert transforms. Singular integral equations. *Levinson*

18.29 ***Intermediate Differential Equations***
Prereq.: 18.21
Year: U (1) 3–0–9
First-order nonlinear differential equations. Second-order linear differential equations. Existence theorems. Successive approximations and Cauchy polygons. Plane autonomous systems. Theory and methods of numerical integration of differential equations. Nonlinear oscillations. (*Not offered 1965–66.*) *Glimm*

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18.292 *Intermediate Differential Equations*
Prereq.: 18.21 or 18.29
Year: U (2) 3–0–9
Regular singular points in the complex domains. Sturm-Liouville systems; special functions; Bessel, Legendre, and related functions. Asymptotic estimates of eigenvalues and eigenfunctions, eigenfunction expansions. Asymptotic integration of linear differential equations. (*Not offered 1965–66.*) *Glimm*

18.31 *Theory of Functions* (A)
Prereq.: 18.21
Year: G (1) 3–0–9
Complex numbers, Riemann sphere, plane topology, fractional linear transformations. Complex integration, Cauchy's theorem, Cauchy's integral formula, power series, winding numbers, rational functions. Singularities, simple Riemann surfaces, local mapping theorem. Evaluation of definite integrals. Integral and meromorphic functions, special functions. Riemann mapping theorem. *Maskit*

18.32 *Theory of Functions* (A)
Prereq.: 18.31
Year: G (2) 3–0–9
Topics selected by the instructor, e.g., harmonic functions, Dirichlet problem, Green functions, conformal mapping, Riemann surfaces, entire and meromorphic functions, Fourier and Laplace transforms, Wiener-Hopf equations, special functions, prime number theorem, elliptic and automorphic functions. *Maskit*

18.33 *Functions of Complex Variables* (A)
Prereq.: 18.31, 18.35
Year: G (1) 3–0–9

18.34 *Functions of Complex Variables* (A)
Prereq.: 18.33
Year: G (2) 3–0–9
Various aspects of the theory of analytic functions. Content varying from year to year, so that students may take the subject in successive years. (*Not offered 1965–66.*) *K. M. Hoffman*

18.35 *Functions of Real Variable* (A)
Prereq.: 18.22
Year: G (1) 3–0–9

18.36 *Functions of Real Variable* (A)
Prereq.: 18.35
Year: G(2) 3–0–9
Elements of the theory of metric spaces, category and Baire theorem. Lebesgue integration, absolute continuity and differentiation of functions of bounded variation. Abstract measure theory, Radon-Nikodym and Fubini theorems, L^P spaces. Elements of Banach spaces, Hahn-Banach and closed graph theorems. Topics on linear transformations on L^P spaces. *Goodman*

18.363 *Abstract Analysis* (A)
Prereq.: 18.25, 18.36
Year: G (1) 3–0–9

18.364 *Abstract Analysis* (A)
Prereq.: 18.363
Year: G (2) 3–0–9
Basic theoretical results about the structures of a mixed analytical-algebraic-geometrical character that are significant in modern analysis. Integration, linear topological spaces and algebras, spectral theory, rings of operators in Hilbert space, group representations. Illustrative applications to harmonic analysis, probability, partial differential equations. (*Not offered 1965–66.*) *Segal*

18.365 *Mathematical Analysis of Fundamental Physics* (A)
Prereq.: 18.36
Year: G (2) 3–0–9
An introductory synthesis of contemporary mathematics and fundamental physics, with particular attention to compactness and precision. Models and methods for the treatment of classical and quantum, discrete and continuum mechanics; relativistic quantum fields and particles; selected non-linear problems. (*Not offered 1965–66.*) *Segal*

18.368 *Hilbert Space* (A)
Prereq.: 18.36
Year: G (2) 3–0–9
General theory of bounded and unbounded operators in Hilbert space, and algebras of such. Illustrative applications, as time permits, to partial differential equations, harmonic analysis, and to quantum mechanics. (*Not offered 1965–66.*) *Segal*

18.37 *Functional Analysis* (A)
Prereq.: 18.27 or 18.31; 18.35
Year: G (1) 3–0–9
General theory of linear topological (especially Banach) spaces and algebras. Spectral theory, especially in Hilbert space; applications to partial differential equations. Structure (multiplicity) theory for abelian operator rings. Harmonic analysis on groups; transformation groups and induced representations. *Segal*

18.38 *Functional Analysis* (A)
Prereq.: 18.37
Year: G (2) 3–0–9
Introduction to the general theory of partial differential equations; weak solutions, the Soboleff inequalities, non-linear equations. Generalized functions (including distributions); Fourier analysis in the complex domain. Applications to analysis on groups and manifolds; ergodic theory. Representations of continuous groups, especially Lie groups. C*-algebras; rings of operators. *Segal*

18.41 *Differential Equations* (A)
Prereq.: 18.15 or 18.20T; 18.31
Year: G (1) 3–0–9

18.42 *Differential Equations* (A)
Prereq.: 18.41
Year: G (2) 3–0–9
Existence theorems in real and complex cases. Linear systems. Regular singular points. Irregular singular points and asymptotic series. Boundary value problems including the non-self adjoint case and the case of infinite intervals. Stability theory and perturbation theory for non-linear differential equations. Periodic orbits. Limit cycles. Differential equation on a torus. (*Not offered 1965–66.*) *Levinson*

18.43 *Partial Differential Equations* (A)
Prereq.: 18.22
Year: G (1) 3–0–9

18.44 *Partial Differential Equations* (A)
Prereq.: 18.43
Year: G (2) 3–0–9
The Cauchy-Kowalewski theorem, existence and regularity of solutions of the Dirichlet problem for linear elliptic equations, existence and regularity of solutions of the Cauchy problem for hyperbolic linear equations, fundamental solutions for linear equations with constant coefficients. *Kotake*

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18.441 *Advanced Partial Differential Equations* (A)
Prereq.: 18.36
Year: G (1) 3–0–9

18.442 *Advanced Partial Differential Equations* (A)
Prereq.: 18.441
Year: G (2) 3–0–9

Regularity properties of solutions of elliptic boundary value problems; spectral theory of elliptic and hypoelliptic differential operators; applications to initial value problems for parabolic and hyperbolic equations. (*Not offered 1965–66.*) ——

18.49 *Harmonic Analysis and Potential Theory*
Prereq.: 18.21
Year: U (2) 3–0–9

Elementary theory of Fourier series and Fourier integrals; Riesz-Fischer and Plancherel theorems; Fourier-Stieljes transforms; potentials of mass distributions; harmonic functions; the Dirichlet problem. *McKean*

18.53 *Theory of Probability* (A)
Prereq.: 18.22 or 18.35
Year: G (1) 3–0–9

18.54 *Theory of Probability* (A)
Prereq.: 18.53
Year: G (2) 3–0–9

Probability measures, Borel fields, conditional probabilities, sums of independent random variables, limit theorems, zero-one laws, martingales, and topics selected by the instructor, for example, information theory, Markov processes, stationary processes and their connections with other parts of mathematics, especially with differential and integral equations. (*18.54 not offered 1965–66.*) *McKean*

18.57 *Theory of Approximation Methods* (A)
Prereq.: 18.20T or 18.26
Year: G (1) 3–0–9

Analysis of constructive methods for the solution of algebraic and differential equations and eigenvalue problems. Applications of functional analysis. Constructive theory of functions. Over-relaxation and other iterative schemes for large systems of difference equations. *Strang*

18.58 *Partial Difference Equations* (A)
Prereq.: 18.20T or 18.26
Year: G (2) 3–0–9

Parallel treatment of initial-value problems for partial differential equations and their finite difference analogues; equivalence of convergence and stability for linear systems; von Neumann's method of stability analysis in L_2 by Fourier transform; Lax theory of nonlinear conservation laws. *Strang*

18.60T *Introduction to Fluid Mechanics*
Prereq.: 18.05T or *18.651*
Year: U (1) 3–0–9

Elements of the theory of perfect fluids; fundamental concepts and basic relations. Theory of potential motions. Motion of rectilinear vortices. Gravity waves and related free surface problems. Application of the theory of complex variables. Elements of the theory of viscous fluids. Some exact solutions. Low-speed flow. Boundary layer theory. *Pedlosky*

18.61 *Introduction to Geophysical Fluid Dynamics* (A)
Prereq.: 18.60T
Year: G (2) 3–0–9

Introduction to the theories of geophysical fluid dynamics. Waves in rotating systems. Concept of potential vorticity. Large scale atmospheric motions. Long atmospheric waves and instability waves in the atmosphere. The theory of quasi-geostrophic motions, and related topics. *Pedlosky*

18.62 *Hydrodynamic Stability and Turbulence* (A)
Prereq.: 18.60T
Year: G (2) 3–0–9

Theory of turbulent motion. Review of the basic equations of hydrodynamics. Statistical theory of turbulence, kinematics, correlation, and spectral representations. Dynamics of homogeneous turbulence, similarity concepts, Kolmogoroff's theory, Heisenberg's theories, the quasi-Gaussian approximation. Turbulent diffusion and transfer, molecular theory of diffusion, the problems of random walk and Brownian motion, diffusion and dispersion by continuous movements, empirical methods. Introduction to the theory of hydrodynamic stability. *Stuart*

18.63 *Mathematical Theory of Magneto-Fluid Mechanics* (A)
Prereq.: 18.60T
Year: G (1) 3–0–9

General introduction. Hydromagnetics: basic equations, kinematic problems, magnetohydrostatics. Pipe flow. Vorticity propagation, linear Alfvén waves, Rayleigh's problem. Two-dimensional flows. Classification of problems. Magneto-gasdynamics: two-fluid model and basic equations. One-dimensional flows, two-dimensional (steady) flows. Small amplitude plane waves, simple waves, shocks, pipe flows. Formulation of equations from the Boltzmann equation, general review of plasma physics. *Todd*

18.634 *Topics in Applied Mathematics* (A)
Prereq.: 18.60T and 18.653
Year: G (1) 3–0–9

Varying content from year to year so that students may take subject in successive years. General theory of rotating fluid motions; transient flows; effects of viscosity, stratification, and compressibility; nonlinear interactions; wave motions; applications. *Greenspan*

18.636 *Stellar Dynamics and Galactic Structure* (A)
Prereq.: 8.711; 18.06T or 18.652
Year: G (2) 3–0–9

Types and compositions of galaxies; mass distributions and kinematics. General dynamics of stellar systems: Liouville and Boltzmann equations, isolating integrals; encounters, relaxation times; epicyclic stellar orbits. Dynamics of interstellar material: observations; hydromagnetic equations; gravitational stability of a gas. Theory of star clusters and elliptical galaxies. Gravitational stability of a disk-shaped galaxy; spiral structure. *Toomre*

18.65T *Integral Equations* (A *except XVIII*)
Prereq.: 18.034 or 18.04
Year: G (2) 3–0–9

Objectives: (1) to show how to express physical problems in the form of integral equations, (2) to provide a treatment of the theory of integral equations, and (3) to describe the methods available for solving such equations. Applications to problems arising in several branches of physics and engineering. *Crout*

18.651 *Introduction to Applied Mathematics*
Prereq.: 18.20T or 18.25 and 18.26; *18.21*
Year: U (1) 3–0–9

Interdependence of mathematics and scientific problems, examples; linear and nonlinear problems, deterministic and random processes, Brownian motion and random walk. Scalars, vectors, and tensors; kinematics of a deformable body; functions of several variables; integral theorems. Potential theory, Gauss theorem, equations of Laplace and Poisson, potential of various kinds of distributions. Green's function, minimum principle. Partial differential equations of mathematical physics, solution under prescribed conditions, diffraction of waves, other examples. Jeffreys and Jeffreys, *Methods of Mathematical Physics*, 3d edition. *L. N. Howard*

18.652 *Introduction to Applied Mathematics*
Prereq.: 18.651
Year: U (2) 3–0–9

Partial differential equations of mathematical physics, further examples. Fourier series, completeness, Weierstrass theorem, Fourier integral. The general Sturm-Liouville problem, examples; integral equation. Calculus of variations, Lagrange's equations. The maximum-minimum principle of the characteristic value problem, existence and completeness of characteristic functions, Hilbert space. Courant-Hilbert, Vol. I, *Methods of Mathematical Physics.* *L. N. Howard*

18.653 *Methods of Applied Mathematics I* (A)
Prereq.: 18.27
Year: G (1) 3–0–9

18.654 *Methods of Applied Mathematics II* (A)
Prereq.: 18.28, 18.653
Year: G (2) 3–0–9

General mathematical theory of partial differential equations and methods of solution with applications to physical problems. Topics include single equations of the first order; characteristic surfaces and the classification of equations of higher order and systems of equations; properties of hyperbolic, parabolic, and elliptic equations, boundary conditions and well posed problems, the application of integral transform and other methods to their solution. Asymptotic approximations. Integral equations; Fredholm and Hilbert-Schmidt theory, singular integral equations, application of integral transforms including Wiener-Hopf method, dual integral equations, Riemann problem. Calculus of variations. *Hunter*

18.655 *Methods of Applied Mathematics III* (A)
Prereq.: 18.652 or 18.653
Year: G (1) 3–0–9

Nonlinear problems, perturbation theory, boundary layer theory, expansion methods; similarity solutions, dimensional analysis, nonlinear ordinary and partial differential equations; theory of characteristics, wave motion, group velocity; special topics. *Benney*

18.66T *Introduction to Elasticity*
Prereq.: 18.05T or 18.651
Year: U (2) 3–0–9

Elements of stress and strain in two and three dimensions. Hooke's Law, including thermal effects. St. Venant torsion theory. Problems of the elastic half space. (*Not offered 1965–66.*) *Reissner*

18.67 *Theoretical and Applied Elasticity* (A)
Prereq.: 2.08 or 18.66T; 18.06T
Year: G (1) 3–0–9

18.68 *Theoretical and Applied Elasticity* (A)
Prereq.: 18.67
Year: G (2) 3–0–9

Analysis of stress and strain in three dimensions. Stress-strain relations. Variational principles. Parametric expansion methods. Theories of beams, plates and shells. *Reissner*

18.681 *Theoretical and Applied Elasticity* (A)
Prereq.: 18.68
Year: G (1) 3–0–9

Special topics in the linear and non-linear theory of beams, plates and shells. (*Not offered 1965–66.*) *Reissner*

18.69 *Tensor Calculus* (A)
Prereq.: 18.15 or 18.20T
Year: G (2) 3–0–9

Classical theory of tensors and Riemannian geometry. Applications to mechanics and general relativity. (*Not offered 1965–66.*) *L. N. Howard*

18.721 *Algebraic Geometry* (A)
Prereq.: 18.25
Year: G (1) 3–0–9

18.722 *Algebraic Geometry* (A)
Prereq.: 18.721
Year: G (1) 3–0–9

Introduction to contemporary algebraic geometry. Content varying from year to year with topics selected from the following: elementary properties of algebraic varieties over arbitrary ground fields, linear systems, Riemann-Roch theorem for curves and surfaces; algebraic groups and abelian varieties, complex toruses and theta functions; equivalence theories, rational equivalence ring, Chern classes, generalized Riemann-Roch theorem; Grothendieck schemes, sheaves, cohomology of varieties and schemes in various senses, holomorphic functions, Hilbert schemes, Picard scheme, fundamental group. (*Not offered 1965–66.*) *Mattuck*

18.731 *Algebra* (A)
Prereq.: 18.20T or 18.26; 18.25
Year: G (1) 3–0–9

18.732 *Algebra* (A)
Prereq.: 18.731
Year: G (2) 3–0–9

Theory of fields and their extensions: algebraic and transcendental extensions, Galois extensions, algebraic closures. Structure of groups and rings. Additional topics varying yearly. *Artin*

18.74 *Topics in Algebra* (A)
Prereq.: 18.731
Year: G (2) 3–0–9

Content varies yearly so that graduate students may take the subject in successive years. Topics in the past few years have included algebraic groups, noncommutative rings and algebras, Lie algebras, local rings, nonassociative algebras. *Schafer*

18.763 *Algebraic Number Theory* (A)
Prereq.: 18.731
Year: G (1) 3–0–9

18.764 *Algebraic Number Theory* (A)
Prereq.: 18.763
Year: G (2) 3–0–9

Study of algebraic number fields and function fields, local and global, with emphasis on the sheaf-like viewpoints which generalize to algebraic groups and on cohomological methods. Ideals, ideles, and adeles; valuation theory. Splitting laws, group of units, class numbers, brief discussion of analytic methods. Cohomology of groups and class field theory for number fields and algebraic curves. *Iwasawa*

18.77 *Theory of Numbers*
Prereq.: 18.20T or *18.25; 18.21*
Year: U (1) 3–0–9

Study of primes, congruences, and arithmetic functions and proofs of their asymptotic formulae. Approximations of the real numbers by rationals, Kronecker's theorem, and the introduction of geometry of numbers. Quadratic forms and quadratic number fields. Elementary proof of the prime number theorem. *Roth*

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18.78 ***Topics in the Theory of Numbers*** **(A)**
Prereq.: 18.20T or 18.25; 18.21
Year: G (2) 3–0–9

18.79 ***Topics in the Theory of Numbers*** **(A)**
Prereq.: 18.78
Year: G (2) 3–0–9

Discussion of the Riemann Zeta function. Proof of the prime number theorem through contour integration, the method of contour integration applied to the divisor problem, the solution of Waring's problem via the circle method of Hardy-Littlewood and Vinogradoff's improvements, other applications of the circle method, the general sieve method of Selberg with application to primes, twin primes, Goldbach's problem. Proofs of certain best possible results of the sieve. Content varying from year to year, so that graduate students may take the subject in successive years. (*18.79 not offered 1965–66.*) *Roth*

18.80 ***Topics in Foundations of Mathematics*** **(A)**
Prereq.: 18.20T or 18.25 or 18.886T or *18.89*
Year: G (1) 3–0–9

Theory of recursive functions with applications to logic and set theory. (*Not offered 1965–66.*) *H. Rogers*

18.81 ***Topology*** **(A)**
Prereq.: 18.241, 18.25
Year: G (1) 3–0–9

18.82 ***Topology*** **(A)**
Prereq.: 18.81
Year: G (2) 3–0–9

Fundamental group, covering spaces, simplicial homology, simplicial approximation, manifolds. Homology and cohomology of topological spaces, universal coefficient theorem, plus additional topics to be chosen by the instructor. *Kan*

18.83 ***Advanced Topology*** **(A)**
Prereq.: 18.82
Year: G (2) 3–0–9

Content varying from term to term so that graduate students taking the subject in successive terms may have an introduction to several important phases of topology such as homotopy theory, cohomology theory, fibre spaces, K-theory, combinatorial topology, and/or differential topology. *Anderson*

18.84 ***Locally Compact Groups*** **(A)**
Prereq.: 18.35 or 18.37
Year: G (2) 3–0–9

Haar measure, group algebras, unitary representations, harmonic analysis on compact and abelian groups, Plancherel theorem, duality theorem. (*Not offered 1965–66.*) *Iwasawa*

18.85 ***Lie Groups and Lie Algebras*** **(A)**
Prereq.: 18.22, 18.26
Year: G (1) 3–0–9

Lie groups and Lie algebras over complete fields; structure and representation theory. *Kostant*

18.86 ***Unitary Representations of Continuous Groups*** **(A)**
Prereq.: 18.85
Year: G (2) 3–0–9

Kirilov theory for nilpotent groups over the reals. Theory of Gelfand and Harish-Chandra for semi-simple groups. General approach using a new quantitization theory as applied to symplectic homogeneous spaces. *Kostant*

18.861 ***Representations of Lie Groups*** **(A)**
Prereq.: 18.85
Year: G (2) 3–0–9

Kirilov theory for nilpotent groups, theory of Gelfand and Harish-Chandra for semi-simple groups. General approach using a new quantitization theory as applied to symplectic homogeneous spaces. *Kostant*

18.87 ***Geometry of Manifolds*** **(A)**
Prereq.: 18.22, *18.81*
Year: G (1) 3–0–9

18.88 ***Geometry of Manifolds*** **(A)**
Prereq.: 18.87
Year: G (2) 3–0–9

Real and complex manifolds, Riemannian and Hermitian manifolds, differential forms, connexions and their curvature forms. Study of global properties of complete Riemannian manifolds under various curvature assumptions. The Gauss-Bonnet theorem, the Weil homomorphism, and characteristic cohomology classes. The Hodge theorem on harmonic forms, and its application to Kähler manifolds. Morse theory for Riemannian manifolds. (*18.88 not offered 1965–66.*) *Quillen*

18.881 ***Elliptic Operators***
Prereq.: 18.22, 18.242, *18.87*
Year: G (1) 3–0–9

18.882 ***Elliptic Operators***
Prereq.: 18.881
Year: G (2) 3–0–9

Local theory of elliptic differential and singular integral operators, extension to vector bundles over manifolds, coercive boundary value problems, K(x), index theorem and applications, elliptic complexes, fixed point theorem, Spencer resolution. *I. M. Singer*

18.886T ***Introduction to Mathematical Logic***
Prereq.: 18.20T or 18.25
Year: U (2) 3–0–9

Formal systems of mathematical logic. Propositional calculus. Quantification theory. Models, validity, and semantic implication. Completeness theorem, Skolem-Löwenheim theory, proof procedures. Axiomatic theories, e.g. set theory. Brief treatment of undecidability and incompleteness of axiomatic theories. *Van Dalen*

18.89 ***Mathematical Logic*** **(A)**
Prereq.: 18.20T or 18.25
Year: G (1) 3–0–9

Classical systems of logic; their use in the formalization of mathematical argument. Algebraic structure in logical systems. Brief study of recursive functions, Gödel incompleteness theorem, related topics. *Lopez-Escobar*

18.892 ***Theory of Models*** **(A)**
Prereq.: 18.886T or *18.89* or 21.641
Year: G (1) 3–0–9

Results and methods from the theory of models. Applications to logic and to algebra. Topics on the borderline between logic and algebra. Non-standard analysis. *H. Rogers*

18.893 ***Set Theory*** **(A)**
Prereq.: 18.80 or 18.886T or *18.89* or 21.641
Year: G (2) 3–0–9

Formalization of mathematics in axiomatic set theory. Cardinal and ordinal numbers. Consistency and independence results. *Tharp*

18.894 ***Proof Theory*** **(A)**
Prereq.: *18.886T* or *18.89* or 21.641
Year: G (2) 3–0–9

Consistency and incompleteness. Positive results of Gentzen and his school. Negative results of Gödel. Constructive interpretation of classical results. (*Not offered 1965–66.*) *H. Rogers*

18.90 ***Mathematical Reading*** **(A)**
Prereq.: ——
Year: G (1 or 2) *Arr.*

Reading of advanced mathematical treatises under the supervision of some member of the department. Choice of treatise and allotment of time according to individual cases. For graduate students finding it desirable to do

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advanced work not provided for in the regular subjects. Open to undergraduates only under exceptional circumstances. *W. T. Martin*

18.91 ***Seminar in Mathematics* (A)**
Prereq.: 18.06T or 18.22 or 18.31
Year: G (1 or 2) *Arr.*
Reading, consultation and discussion of current mathematical research. (*Not offered 1965–66.*) *Levinson*

19.

Meteorology

19.003 ***Elementary Meteorology I***
Prereq.: ——
Year: 1 (2) 2–0–4
Introduction to meteorological science through examples from all scales of atmospheric motions; radiation balance and the global circulation; air masses and fronts; cyclones and weather forecasting; hurricanes; sea breeze; thunderstorms; micrometeorology; cloud physics; climatology. Laboratory exercises with weather maps. *J. M. Austin*

19.02 ***Descriptive Meteorology***
Prereq.: ——
Year: U (2) 3–0–6
The general circulation of the earth's atmosphere; the secondary circulations, both tropical and extratropical, including the role of air masses and fronts; the tertiary (local) circulations, including drainage winds, local convection, whirlwinds, showers, thunderstorms, land and sea breeze, large scale turbulence eddies, föhn winds, etc. Willett and Sanders, *Descriptive Meteorology*, 2nd ed. *Willett*

19.14 ***Long-Range Weather Forecasting* (A)**
Prereq.: 19.45T
Year: G (1) 3–8–6
Physical and statistical analysis of large-scale changes of the general circulation, with emphasis on the variable solar influences and climatic trends. Review of extended and long-range forecasting techniques. Laboratory exercises on the application of the Weather Bureau five-day prognostic technique. *Willett*

19.15 ***Long-Range Forecasting* (A)**
Prereq.: 19.45T
Year: G (1) 3–0–6
Lecture part of 19.14. *Willett*

19.22 ***Air and Sea Instruments* (A)**
Prereq.: 12.811T or 19.83; 19.43T
Year: G (2) 3–2–6
Lectures and laboratory exercises on the design and response characteristics of typical instruments and instrument systems used for measurements in the atmosphere, in the oceans, and in the sea-air interfacial region. *Keily*

19.24 ***Fluid Dynamics Laboratory* (A)**
Prereq.: 1.612 or 2.201T or 16.041 or 18.60T or 19.62
Year: G (2) 2–4–4
Experiments on processes and phenomena in geophysical fluid mechanics. Design of experiments, similarity, experimental methods, data processing. Laboratory work involving quantitative observation of hydrodynamic instability, transport processes and turbulence. *Mollö-Christensen*

19.34 ***Statistical Methods in Meteorology***
Prereq.: 18.034 or 18.04; *19.51*
Year: G (1) 2–0–4
Designed to examine recent developments in statistical weather forecasting and to provide a background for performing and evaluating statistical studies with meteorological data. Statistical forecasting by linear regression, nonlinear statistical forecasting, significance of empirical relations, and hypothesis testing. *Lorenz*

19.35 ***Statistical Problems in Meteorology* (A)**
Prereq.: 19.34
Year: G (2) 2–0–4
Seminar in statistical problems in meteorology, including such topics as power spectrum analysis, properties of stationary time series, and evaluation of linear and nonlinear methods of statistical weather forecasting. *Lorenz*

19.43T ***Synoptic Meteorology I***
Prereq.: 8.02T, 18.02T
Year: U (1) 2–2–4
The state of the atmosphere including the three-dimensional variation of significant meteorological variables. The heat budget of the atmosphere on a global and local scale. The physical processes associated with the formation of fog, smog, cumulus clouds, thunderstorms, and tornadoes. Introduction to the dynamics of cyclones and hurricanes. World and local climate. Petterssen, *Weather Analysis and Forecasting*, 2nd Edition, Vol. II. *J. M. Austin*

19.45T ***Synoptic Meteorology II* (A)**
Prereq.: 19.43T
Year: G (2) 3–0–6
Historical development of weather prediction. Application of elementary kinematics and dynamics to patterns of atmospheric flow. Description of structure and behavior of cyclonic and anticyclonic circulations. Simple dynamical prediction models. Petterssen, *Weather Analysis and Forecasting*, Vol. I, 2nd ed. *Sanders*

19.46 ***Numerical Weather Prediction* (A)**
Prereq.: 18.05T
Year: G (2) 3–0–6
Scale analysis of atmospheric motions and the problem of meteorological noise. The quasi-geostrophic forecast system. Finite difference methods and the analysis of computational stability. Derivation and study of some simple atmospheric models for numerical forecasting. *Phillips*

19.51 ***Synoptic Laboratory I***
Prereq.: 19.02
Year: U (1) 0–15–0
Techniques of analysis and prognosis of the daily circulation pattern. Dynamical and empirical methods for prediction of the circulation pattern and associated weather phenomena. *Sanders*

19.53T ***Synoptic Laboratory II* (A)**
Prereq.: 19.43T
Year: G (2) 0–4–8
Study of atmospheric phenomena in the troposphere and stratosphere by analysis of meteorological data. Consideration of scales ranging from the global general circulation to severe thunderstorm complexes. Data sources include the global network of surface and upper-level balloon sounding stations on land and at sea, aircraft data, rocket soundings in the stratosphere, weather radar, and meteorological satellites. Weather prediction as a probabalistic science. Contemporary operational forecasting as a man-machine system. *Sanders*

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