George Proctor Wadsworth, Ph.D.



### Undergraduate Study

#### C. L. E. Moore Instructors

Allan Russell Adler, Ph.D. Gerald Arthur Anderson, Ph.D. Michael Walter Davis, Ph.D. Josef Dodziuk, Ph.D. Harold Gerald Donnelly, Ph.D. Henryk Hecht, Ph.D. Solomon Jekel, Ph.D. Edward Yarnell Miller, Ph.D James Richard Wason, Ph.D.

#### Instructors

Arnold Irvin Barnett, Ph.D. Gail Alexandra Carpenter, Ph.D. Darell James Johnson, Ph.D. Zvi Meir Kedem, Ph.D. Nelly Sebilla Kroonenberg, Ph.D. Robert Arthur LaBudde, Ph.D. Yue-Ying Lau, Ph.D. Philip William Lavori, Ph.D. Anne Marie Leggett, Ph.D. Shuo-Yen Robert Li, Ph.D. Julian Ivanhoe Palmore III, Ph.D. Steven Robbins, Ph.D. Philip Trauber, Ph.D. Da-Lun Wang, Ph.D.

#### Administrative Assistant

James Edward Dalton

### Rèsearch Associate

Herm Jan Brascamp, Ph.D

#### Professors Emeriti

Prescott Durand Crout, Ph.D. Professor of Mathematics, Emeritus Senior Lecturer

Raymond Donald Douglass, Ph.D., Sc.D. Professor of Mathematics, Emeritus

Professor of Mathematics, Emeritus

Professor of Mathematics, Emeritus

### **Bachelor of Science in Mathematics** Course XVIII

Each undergraduate's program is arranged through continuous collaboration between the individual student and his or her faculty counselor, subject to a minimum set of specific requirements. In most cases, the undergraduate curriculum in mathematics is preparatory to further professional training at the graduate level. For this purpose students are encouraged both to obtain a substantial grounding in each of the fundamental branches of mathematics and to explore certain fields of application in order to establish a basis for the selection of an appropriate field of graduate specializa-

In light of students' career objectives teaching, participation in research programs of an industrial or government sponsored organization, or work as a consultant in a business or a modern high-speed computation center - the immediate educational aim of the Department is to provide both an understanding of a substantial part of the existing body of mathematical knowledge and an ability to impart this knowledge to others. Most importantly, however, the Department hopes to inspire a deep interest in the discovery or invention of new mathematics or in the application of mathematics to a new field.

Undergraduates wishing to work in small groups under the supervision of a faculty member or to do individual work with a member of the Department may elect either to participate in a mathematics seminar or to write a thesis. The experience gained from active participation in a seminar conducted by a research mathematician may be particularly valuable for a student planning to pursue graduate work in some branch of mathematics.

The requirements are unspecific in order to accommodate those students who may not be professionally interested in mathematics but who wish to use it in connection with other work, and to give professionally oriented students maximum flexibility in exploring their interests in both pure and applied mathematics.

### Bachelor of Science in Mathematics Course XVIII

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900	General Institute Requirements	Total u	nits
	Specific science and humanities and social science subjects		132
	The Science Distribution Requirement will be satisfied by 18.03 or 18.031, which are among the Restricted Electives, plus appropriate subjects totaling		24
	Laboratory Requirement		12
	Departmental Program		
	Restricted Electives One of the following subjects:		108
18.03	Differential Equations	4 0 8	
18.031	Introduction to Linear Algebra and Differential Equations	4 0 8	
	Course XVIII subjects of essentially different content, including at least six with the first decimal digit one or higher, and excluding 18.05, 18.051 and 6-unit electives.	96	
	Unrestricted Electives		84
	Total Units Required for S.B. Degr	86	360

Advanced standing credit will not fulfill any part of this requirement, but will be accepted as Unrestricted Elective

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M.I.T.

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Students who expect to pursue a career in pure mathematics should normally include in their program a year of analysis selected from 18.100 Analysis I, 18.101 Analysis II, 18.104 Seminar in Analysis, and 18.115 Functions of a Complex Variable or 18.284 Introduction to Functions of a Complex Variable.

units

132

24

12

They should also usually take a year of algebra in one of the sequences 18.701 Algebra I and 18.702 Algebra II, or 18.700 (or 18.710) Linear Algebra and 18.703 Modern Algebra, as well as 18.901 Introduction to Topology. The remaining subjects may be chosen from such areas as logic, number theory, differential topology, combinatorial theory, probability and statistics, or other applied mathematics subjects.

Professionally oriented pure mathematics students are strongly urged to plan their programs so that at least five of the upper-level subjects are completed by the end of the third year. It is also recommended that the overall program include at least one or two graduate-level subjects and a seminar. The faculty counselor can make useful suggestions concerning both the subjects themselves and the order in which students should take them, according to individual interests and abilities.

Applied mathematics is the mathematical study of general scientific concepts, principles, and phenomena which, because of their widespread occurrence and application, relate or unify various disciplines. The core of the program at M.I.T. concerns the following principles and their mathematical formulations: propagation, equilibrium, stability, optimization, cybernetics, statistics, and random processes. The following sequence of subjects is recommended because it provides a general introduction to most areas of applied mathematics and to several specific areas for study in greater depth.

Freshmen interested in applied mathematics should take 18.001 Calculus and 18.002 Calculus followed as soon as possible by 18.03 Differential Equations or 18.031 Introduction to Linear Algebra and Differential Equations. 18.03 and 18.002 may be taken during the same term.

Sophomores should survey the field of applied mathematics by enrolling in both 18.041 Principles of Applied Mathematics and 18.042 Principles of Applied Mathematics. 18.041, given in the fall term only, is devoted to the discrete aspects of the subject and may be taken concurrently with 18.03.

Juniors should deepen their understanding of general concepts and techniques by acquiring more intensive specialization in one of the following areas: 18.301 Introduction to Physical Mathematics I, 18.312 Statistics for Applications, 18.313 Probability (or 18.303 Probability and Random Variables and 18.304 Statistical Inference), 18.401 Control Theory, and 18.420 Introduction to the Mathematical Theory of Computation.

Subjects of equivalent content may be substituted for these. For further study in the junior or senior years, students should also consider 18.284 Introduction to Functions of a Complex Variable, 18.302 Introduction to Physical Mathematics II, 18.314 Applied Combinatorial Analysis, 18.330 Introduction to Elasticity, 18.350 Introduction to Fluid Mechanics, 18.370 Introduction to Dynamics in Astronomy and 18.411 Applied Algebra.

Sample programs emphasizing different specialities — continuum mechanics, combinatorics, numerical analysis, etc. — are available in the Undergraduate Mathematics office.

Various mixed strategies combining pure and applied mathematics subjects are possible. All students are encouraged to acquire as broad a background in both general areas as time and their inclinations will allow. The great majority of mathematics majors do in fact take both pure and applied subjects.

Students who wish to major in mathematics but do not intend to make a career of it are welcome in any of the above programs.

### Inquiries

Any additional information regarding academic programs, admissions, financial aid, etc. may be obtained from the Undergraduate Mathematics Office, 2-108, M.I.T., Cambridge, Massachusetts 02139.

**Mathematics** 

### **General Mathematics**

#### 18.U.R. Undergraduate Research

Prereq.: -Year: U (1, 2)

Undergraduate research opportunities in Mathematics. For further information consult the Departmental Coordinator: C. M. Bender. Staff

#### 18.001 Calculus

Prereq.: Year: U (1) 5-0-7

Emphasis on the concepts and techniques ofcalculus relevant to science and technology. Mathematical formulation of problems and approximate methods of solution. Limits, differentiation, rates, maxima and minima problems, applications, Taylor series, L'Hôpital's rule, finite differences, definite and indefinite integrals, techniques for integration, improper integrals, the Monte-Carlo method, perturbation and iteration procedures, stability, series, summation techniques, asymptotic series, symbolic methods, numerical analysis, differential equations. G.-C. Rota, P. S-H. Wang

### 18.002 Calculus

Prereq.: 18.001 or 18.01

Year: U (1, 2) 5-0-7

Continuation of 18.001. Presentation of the concepts and techniques of calculus relevant to science. Vector algebra, analytic geometry, planetary motion, orbit stability, partial differentiation, functions of several variables, Taylor series, extremal problems, linear programming examples, numerical methods, multiple integrals, approximate and asymptotic methods of evaluation, applications; vector calculus, gradient, curl, theorems of Stokes, Green & Gauss, conservation laws, fluid motion. Term 1: Val. Toomre Term 2: H. Cheng

#### 18.01 Calculus (Revised: Unit change)

Prereq.: Year: U (1, 2) 5-0-7

Differentiation and integration of functions of one variable, with applications. Concept of function, limits, and continuity. Differentiation rules, application to graphing, rates, approximation, and extremum problems. Mean-value theorem. Definite and indefinite integration. Fundamental theorem of calculus. Applications of integration to geometry and science. Elementary functions. Techniques of integration. Approximation of definite integrals, infinite series, improper integrals, Taylor's formula and l'Hôpital's rule. W. T. Martin, F. P. Peterson, P. Lavori, G. Anderson

### 18.02 Calculus (Revised: Unit change)

Prereq.: 18.001 or 18.01 Year: U (1, 2, S) 5-0-7

Functions of several variables, with elementary linear algebra. Vector algebra in 3-space and n-space. Matrices, row-reduction, systems of linear equations. Vector-valued functions of one variable; space motion. Scala functions of two and three variables; partial differentiation, gradient, approximation techniques. Double and triple integration, with applications. Vector fields, line integrals, exact differentials. Green's theorem, Divergence theorem. Stokes' theorem.

Term 1: A. P. Mattuck, H. Rogers, Jr. Term 2: Staff

### 18.023 Topics in Calculus

Prereg.: 18.002 or 18.02 Year: U (2) 2-0-4

Primarily for students completing 18.02 during the first half of the second term. Supplements the calculus sequence. Topics to be announced. (Not offered 1975-76.) A. P. Mattuck

### 18.03 Differential Equations

Prereq.: 18.002 or 18.02 Year: U (1, 2, S) SD 4-0-8

Examples of initial value problems in science and engineering associated with single equations and with systems of first order equations. Methods of solution include graphical constructions, similarity transformations, series, Laplace transforms, matrices, numerical integration and the phase plane. Emphasis on formulation of natural phenomena in terms of differential equations and on interpretation of the solutions.

Term 1: E. M. Kleinberg, F. B. Hildebrand Term 2: A. P. Mattuck, W. G. Strang

## 18,031 Introduction to Linear Algebra and Differential Equations

Prereq.: 18.002 or 18.02 Year: U (1, 2) SD 4-0-8

Vector spaces, bases, linear dependence, inner products; linear transformations and their representation by matrices, structure theory of matrices, eigenvalues, quadratic forms.

Elementary theory of differential equations, emphasizing linear differential equations and systems with constant coefficients. Suitable as background for subjects requiring more linear algebra but less differential equations than 18.03.

Term 1: R. P. Stanley Term 2: J. Mandula

### 18.04 Complex Variables with Applications

Prereq.: 18.03 Year; U (1, 2)

First half: Complex algebra and functions; Cauchy-Riemann equations, analyticity; contour integrals, Cauchy's theorem; Taylor and Laurent series; singularities, residues, evaluation of integrals. Second half: Conformal mapping, potential theory in two dimensions; series solutions of ordinary differential equations; representation of functions by contour integrals, Γ (z), J<sub>o</sub>(z); some Fourier analysis and Laplace transforms. 6 units of credit for first half, 12 units for both. J. W-K. Mark

### 18.041 Principles of Applied Mathematics

Prereq: 18.002 or 18.02 Year: U (1) 3-0-9

An introductory survey of fundamental concepts in applied mathematics: optimization, random process, coding, signal propagation. In this independent half of the complete sequence, the emphasis is on the ideas and topics that relate to a "discrete" mathematical approach: computation, combinatorics, probability, linear programming.

### 18.042 Principles of Applied Mathematics

Prereq.: 18.03 or 18.031 Year: U (2) 3-0-9

An introductory survey of fundamental concepts in applied mathematics: propagation, stability, equilibrium, optimization. In this independent half of the complete sequence, the emphasis is on the ideas and topics that relate to a "continuous" mathematical approach, but connection with discrete mathematical approach will also be stressed: random walk, diffusion, waves, instabilities, characteristics and first order partial differential equations: with applications to traffic problems, fluid flow, and other problems in classical mathematical physics.

H. P. Greenspan

# 18.05 Introduction to Probability and Statistics

Prereq.: 18.001 or 18.01 Year: U (2) SD 3-0-9

An elementary introduction, with applications in the social and biological sciences. Less mathematical than 18.443 and 18.313, making only limited use of calculus. Relative frequency. Probability models. Binomial experiments. Normal approximation. Descriptive level of significance. Chi-square approximation. Contingency tables. Hypothesis testing. Confidence regions. Random variables. Distribution-free methods. Mean and variance. Elements of decision theory.

H. Rogers, Jr.

### 18.051 Surveys of Higher Mathematics

Prereq.: 18.001 or 18.01 Year: U (2) SD 3-0-9

Survey of a limited number of topics of current interest and of broad cultural significance, designed for students not majoring in mathematics, and stressing the central role of mathematics in today's world. Topics varying from year to year, and including: current developments in logic, the evolution of geometry, contemporary views on space and time, discrete vs. continuous mathematics, the mathematics of communication, chance and choice, elements of group theory, transfinite arithmetic, elements of field theory, non-standard analysis, etc. (Not offered 1975–76.) G.-C. Rota

### 18.052 Philosophy of Mathematics and of Natural Science (New)

Prereq.: 18.02 or 18.002 Year: U (2) 3-0-9

The unreasonable effectiveness of mathematics in the natural sciences. Case studies of the process of mathematization of scientific theories, stressing conceptual origins. Topics varying from year to year: set theory and its limitations, infinity, evolution of geometrical concepts, the problem of time, the logic of quantum physics, mathematical problems in biology, formalization in science, intentionality and meaning, self-evidence and truth. Based in part on the writings of E. Husserl and H. Weyl.

# 18.075 Advanced Calculus for Engineers (A. except Courses II, VI, VIII, XII, XIII, XVI, XVIII, XXII)

Prereq.: 18.03 or 18.031 Year: G (1, 2, S) 3-0-9

Functions of a complex variable; calculus of residues. Ordinary differential equations; integration by power series; Bessel and Legendre functions. Expansion in series of orthogonal functions, including Fourier series. Hildebrand, Advanced Calculus for Applications.

F. B. Hildebrand

### **Analysis**

### 18.076 Advanced Calculus for Engineers (A, except Courses II, VI, XVI, XVIII, XXII)

Prereq.: 18.075 Year: G (1, 2, S) 3-0-9

Vector analysis: orthogonal curvilinear coordinates. Calculus of variations. Solution of classical equations of mathematical physics, including applications of conformal mapping and the Laplace transformation. Partial differential equations; characteristics. Hildebrand, Advanced Calculus for Applications.

F. B. Hildebrand

### 18.085 Methods of Applied Mathematics for Engineers (A, except VI and XVIII)

Prereq.: 18.075 Year: G (1, 2) 3-0-9

Operations with matrices and determinants; linear vector spaces; characteristic-value problems. Techniques of calculus of variations; constraints, direct methods. Formulation and treatment of integral equations; Green's function; analytical and numerical methods of solution. Hildebrand, Methods of Applied Mathematics.

F. B. Hildebrand

### 18.089 Review of Mathematics

Prereq.: Year: G (S)

Review of calculus and differential equations. Primarily for students in Course XIII-A. Degree credit allowed only in special circumstances.)

### 18.093 Tutoring in Mathematics

Prereq.: 18.002 or 18.02 Year: U (1, 2)

For undergraduates who are teaching or tutoring in mathematics subjects. Limited enrollment, based on positions available. Permission must be secured in advance to register for this subject.

A. P. Mattuck

### 18.094 Seminar in Mathematical Education

Prereq.: Year: U (1)

Study of school mathematics from an advanced standpoint together with practical experience in teaching. The seminar will investigate school mathematics curricula and teaching methods in light of modern ideas on teaching intuition and thinking skills. Students required to engage in 4 hours of practical work per week with children. (Enrollment by permission of instructor.) S. A. Papert

### 18.099 Independent Activities

Prereq.: Year: U (1, J, 2) Arr.

For undergraduates desiring credit for studies during I.A.P. or for special individual reading on an undergraduate level during the regular terms. Specific programs and credit arranged in consultation with individual faculty members and subject to departmental approval. Staff

### 18.100 Analysis I (A, except XVIII)

Prereq.: 18.03 or 18.031 Year: U (1, 2) G (1, 2) 3-0-9

Two options offered, both covering the fundamentals of mathematical analysis: real numbers and euclidean n-space; open, closed and connected subsets of n-space; continuous and differentiable functions; convergence of series and sequences of functions. Option A is new, and emphasizes how these fundamental are used in mathematics and its applications to other fields; the utility of abstraction and need for definitions and arguments are made clear, but less emphasis put on students constructing proofs. Option B emphasizes the reading and constructing of rigorous proofs, and includes more abstract concepts than option A (for example, compact sets); it is the "standard" analysis subject. Option A: K. M. Hoffman

Option B: R. M. Dudley and Staff

### 18.101 Analysis II (A, except XVIII)

Prereq.: 18.100 Year: U (1, 2) G (1, 2) 3-0-9

Two options offered, both continuations of 18.100, but in different directions. Option M. (first term) stresses the topics most useful in the study of manifolds and global analysis: differentiable maps, Jacobians, differentials, inverse and implicit function theorems, chang of variables in multiple integration, general Stokes theorem. Option L (second term) stresses classical analysis: Lebesgue integration in euclidean space, its applications to classical Fourier analysis, including the Riesz-Fischer theorem, brief introduction to functional analysis. Much more concrete treatment than in 18.125, 18.126.

Option M: S. Helgason Option L: W. Ambrose

# 18.103 Fourier Analysis—Theory and Applications

New)

Prereq.: 18.03 or 18.031

Year: U (1)

3-0-9

Fourier series and integrals, with applications o science, engineering, and to other branches of mathematics. Theory developed in close elation to the applications. Representation of functions by Fourier series; convergence questions. Solutions of partial differential equations by Fourier methods. Applications to ometry and number theory. Fourier integrals and the heat equation. Plancherel's theorem. Introduction to distributions. Requires less background than 18.101L; in particular, prior experience with theoretical mathematics not required, but students should have an interest n seeing mathematical arguments, as well as some familiarity with complex variables through the residue calculus. N. Kerzman

### 18.104 Seminar in Analysis

Prereq.: 18.100 Year: U (1, 2) 3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussion by students on topics taken from current journals or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite.

### 18.109 The Architecture of Modern Analysis (A)

Prereq.: 18.100 Year: G (2) 3-0-9

An erientation and survey complementary to the grammatical development initiated in the analysis core program. The most central theorems and interconnections of: integration and ergodic theory; linear spaces and harmonic analysis in Euclidean space and on groups; differential equations, Soboleff inequalities, interpolation theory; spectral theory and operator algebra; analysis on manifolds. Some informal proofs and illustrative applications; technical details by reference only. Guest lecturers. (Not offered 1975–76.)

### 18.115 Functions of a Complex Variable (A)

Prereq.: 18.100

Year: G (1)

3-0-9

Cauchy's integral theorem, residue theorem and applications. Series and product representations of analytic functions. Entire and meromorphic functions. Simple Riemann surfaces. Conformal mapping. Additional topics may be included. More theoretical than 18.284

H. M. Stark

### 18.116 Functions of a Complex Variable (A)

Prereq.: 18.115 Year: G (2) 3-0-9

Topics selected by the instructor, e.g., harmonic functions, Dirichlet problem, Riemann surfaces, uniformization, quasi-conformal mapping, Fourier and Laplace transforms, special functions.

A. R. Adler

#### 18.117 Several Complex Variables (A)

Prereq.: 18.115, 18.125

Year: G (1)

3-0-9

### 18.118 Several Complex Variables (A)

Prereq.: 18.117 Year: G (2) 3-0-9

Holomorphic functions of several complex variables. Biholomorphic maps, pseudo-convexity, domains of holomorphy, Dolbeault cohomology, vanishing theorems. Second term: the δ-Neumann problem, PDE-theoretic methods in several complex variables, integral representations.

N. Kezman

### 18.125 Measure and Integration (A)

Prereq.: 18.101 or 18.901

Year: G (1)

3-0-9

Set algebra and measurability; integration and construction of measures; the Fubini theorem; convergence theorems; set functions, euclidean analysis.

I. E. Segal

### 18.126 Functional Analysis (A)

Prereq.: 18.125 Year: G (2) 3-0-9

Topological and normed vector spaces. Hilbert space, Banach algebras, compact operators, distributions and Fourier transforms.

I. E. Segal

#### 18.135 Fourier Analysis (A)

Prereq.: 18.126 Year: G (1) 3-0-9

Fourier transorms and analytic functions, Laplace transorms, the Paley-Wiener theorem, Hilbert transforms, interpolation theory, Lp theory of Fourier transforms. (Not offered 1975–76.)

A. P. Calderon

# 18.155 Introduction to Partial Differential Equations (A)

(New)

Prereq.: 18.101 Year: G (2) 3-0-9

The classical theory of second order linear partial differential equations. Classification as elliptic, parabolic, or hyperbolic; boundary, conditions leading to well posed problems; physical origins of typical equations. Elliptic equations: existence, uniqueness, and regularity for the Dirichlet and Neumann problems; use of complex variables. Parabolic equations: the heat equation. Hyperbolic equations: the Cauchy problem; bicharacteristics and the propagation of singularities. Some first order equations also discussed. More elementary and concrete than 18.157; for graduates and advanced undergraduates. (Alternate years. Not offered 1975-76.) D. G. Schaeffer

### 18.157 Partial Differential Equations I (A)

Prereq.: 18.101 Year: G (1) 3-0-9

Fundamental solutions for linear equations with constant coefficients. Cauchy-Kowalewski theorem, elliptic equations, pseudo-differential operators, boundary-value problems. Some familiarity with functional analysis presumed. Permission of Instructor Required.

W. Ambrose

## Applied Mathematics

### 18.259 Theoretical and Observational Cosmology (A)

Prereq.: Permission of Instructor

G (2)

the implications of causality, covariance, cality, and stability for possible space-time and field-theoretic models. Comparison of specific theories with recent astronomical observations. Maturity in one or more of stronomy, mathematics, or physics at the evel normally indicated by a second-year duate status is assumed. (Not offered 975-76.) I. E. Segal

### 18.275 Numerical Analysis (A, except XVIII)

Prereq:: 18.075 or 18.301

Year: G (1) . 3-0-9

### 8.276 Numerical Analysis (A)

Prereq.: 18.275 Year: G (2) 3-0-9

Introduction to the theory and practice of the solution of equations, numerical differentiation and integration, and the numerical solution of ordinary and partial differential equations. Additional topics from the following: interpolation, eigenvalues and eigenvectors of matrices, efficient summation of series, east-squares methods, smoothing of data, Gaussian quadrature, Chebyshev approximation, harmonic analysis, pproximation by exponential and rational functions and by splines. (18,276: Alternate years: Not offered: 1975-76, when a faster experimental version of 18.275 will touch main topics from both terms.) A. Toomre

### 18.279 Analysis of the Finite Element Method (A)

Prereq.; Permission of Instructor

Year: G (2) 3-0-9

Variational formulation of differential equations. Rayleigh-Ritz-Galerkin technique for steady-state problems. Construction of piecewise polynomial subspaces and their pases. Degree of approximation by polynomials. Rate of convergence of the finite element method. Examples. Eigenvalue, initial-value, and nonlinear problems by Galerkin's method. (Alternate years. Not offered 1975-76.) W. G. Strang

### 18.284 Introduction to Functions of a Complex Variable (A, except XVIII)

Prereq.: 18.03 or 18.031

Year: G (1) 3-0-9

18.285 Introduction to Functions of a

### Complex Variable (A)

Prereq.: 18.115 or 18.284

Year: G (2)

3-0-9

Complex numbers, analytic functions, Riemann surfaces for certain functions, Cauchy's theorem, singularities, residues, contour integrals, conformal mapping, Schwarz-Christoffel transformation, series and sequences, analytic continuation, harmonic functions, conjugate functions, the gamma function, second-order linear differential equations and special functions. Laplace transforms, asymptotic series, saddle-point method, Hilbert transforms. Singular integral equations. More advanced than 18.04. 18.284: C. M. Bender, 18.285: Y. Lau

#### 18.295 Tensor Calculus (A)

Prereq.: 18.031 or 18.085 or 18.700

Year: G (2)

3-0-9

Tensors and Riemannian geometry with emphasis on geometric ideas and applications to mechanics and general relativity. (Alternate years. Not offered 1975-76.) L. N. Howard

### 18.301 Introduction to Physical Mathematics I

Prereq.: 18.03 Year: U (1)

3-0-9

Interdependence of mathematics and scientific problems, examples; deterministic and random processes; particle mechanics and differential equations, Brownian motion and random walk; Fourier analysis; tensors; partial differential equations of mathematical physics and continuum mechanics.

C. C. Lin

#### 18.302 Introduction to Physical Mathematics II

Prereq.: 18.04 or 18.284 or 18.301

Year: U (2)

3-0-9

Partial differential equations of mathematical physics, classification and solution of boundaryvalue problems, method of separation of variables. Sturm-Liouville problems, Fourier integrals, integral transforms. Bessel functions and other special functions. Applications of calculus of variations. (A knowledge of complex variables desirable but not essential.) S. A. Orszag

### 18.305 Methods of Applied Mathematics I (A)

Prereq.: 18.071 or 18.075 or 18.284

Year: G (1)

3-0-9

### 18.306 Methods of Applied Mathematics II (A)

Prereq.: 18.305

Year: G (2)

3-0-9

A comprehensive treatment of the advanced methods of applied mathematics. Term 1: asymptotic behavior of differential and difference equations; asymptotic evaluation of integrals; regular and singular perturbation methods; boundary-layer techniques; WKB method; multiple scales; divergent series. Term 2: partial differential equations; transform methods; characteristics, initial and boundaryvalue problems; Green's functions; numerical methods, singular perturbation problems; nonlinear wave propagation. 18.305: S. A. Orszag, 18.306: C. M. Bender

### 18.158 Partial Differential Equations II (A)

Prereq.: 18.157 Year: G (2) 3-0-9

Hamilton-Jacobi theory, Fourier integral operators, initial-value problem for hyperbolic equations, general existence and uniqueness theorems in terms of characteristics, spectral theory.

W. Ambrose

### 18.165 Pseudodifferential Operators (A)

Prereq.: Permission of Instructor

Year: G (1) 3-0-9

### 18.166 Pseudodifferential Operators (A)

Prereq.: 18.165 Year: G (2) 3-0-9

Distributions, Fourier transforms and H, spaces; definition of pseudodifferential operators, their continuity properties, the Schwartz kernel, and the functional calculus. Elliptic and hypoelliptic pseudodifferential operators. The H<sub>s</sub>, m-spaces, Boundary value problems. Pseudodifferential operators on manifolds and the Atiyah-Singer index theorem. (Not offered 1975–76.)

A. P. Calderon

### 18.175 Theory of Probability (A)

Prereq.: 18.125 Year: G (2) 3-0-9

Survey of contemporary theory of stochastic processes. Independence, zero-one laws, martingales, stopping rules, stationary processes, elements of ergodic theory; Markov chains, characteristic functions, central limit problems, stable laws; renewal theory, Tauberian theory, Gaussian processes and their spectral theory, Brownian motion, the strong Markov property, the variance theorem; Kolmogorov-Smirnov statistics, processes with independent increments, diffusion theory. Topics selected each year from these and others. (May be repeated for credit. Not offered 1975–76.)

### 18.176 Topics in Probability (A)

Prereq.: 18.175 Year: G (1) 3-0-9

Gaussian processes, sample function continuity, equivalence and singularity of processes, the isonormal process, and other topics. (Not offered 1975–76.)

R. M. Dudley

# 18.177 Introduction to Stochastic Processes (A)

Prereq.: 18.313 Year: G (2) 3-0-9

Topics in stochastic processes, including random walks, Poisson and birth-death processes, Brownian motion and diffusion. Emphasis on connections with differential and integral equations. (Not offered 1975–76.)

D. B. Ray

#### 18.194 Seminar in Analysis

Prereq.: 18.100 or 18.301

Year: U (2) 3-0-9

Seminars in several topics, for mathematics majors. Each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussions by students on topics taken from current journals or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite. (Not offered 1975–76.)

L. N. Howard

#### 18.199 Topics in Analysis (A)

Prereq.: Permission of Instructor Year; G (1, 2) 3-0-9

Topics vary from term to term; may be repeated for credit. (Not offered 1975-76.)

# 18.235J The Mathematics of Quantum Mechanics (A)

(Same subject as 8.372J)
Prereq.: Permission of Instructor
Year: G (1)
3-0-9

Topics in elementary analysis and functional analysis that are important for quantum mechanics: linear spaces; topological spaces; measure theory; Hilbert spaces. The subject leads up to the definition of a self-adjoint operator and the spectral decomposition theorem for such operators. It is intended that this subject will supplement and complement the usual subjects in quantum mechanics and mathematical methods by: (1) outlining the mathematical foundations of quantum mechanics: (2) introducing the student to modern mathematical concepts and techniques; (3) providing some background for modern quantum field theory. (Not offered 1975-76.) E. H. Lieb

### 18.238J Modern Statistical Mechanics (A)

(Same subject as 8.336J) Prereq.: 8.334 Year: G (2) 3-0-9

Selected topics, primarily in equilibrium statistical mechanics, with emphasis on rigorous mathematical results. Discussion in detail of exactly soluble models such as the two-dimensional Ising and ferroelectric models. Theories of phase transitions.

K. Hepp

### 18.247 Topics in Operator Theory (A)

Prereq.: 18.126 Year: G (1) 3-0-9

Topics vary from year to year; may be repeated for credit. Recent topics have included the spectral theorem for normal operators, C\*-algebras, von Neumann algebras group representations and semigroups, perturbation and scattering theory. (Not offered 1975–76.)

I. E. Segal

### 18.255 Mathematical Methods of Advanced Physical Theory (A)

Prereq.: 18.125 Year: G (1) 3-0-9

Compact rigorous methods which have found application in physical field and particle theory. Spectral theory in Hilbert space; perturbation and scattering theory; evolutionary partial differential equations. Group representations in Hilbert space; operator algebra; functional integration. Examples from relativistic wave equations and quantum field theory. (Not offered 1975–76.) I. E. Segal

### 18,257 Mathematical Theory of Quantum Fields (A)

Prereq.: 18.125 Year: G (2) 3-0-9

The general theory of quantized relativistic fields. Introduction to the problem of the construction of nonlinear fields. (Not offered 1975–76.)

I. E. Segal

### 18.307 Methods of Applied Mathematics III (A)

Prereq.: 18.302 or 18.305

Year: G (1) 3-0-9

Calculus of variations; Integral equations (Volterra equations, Fredholm equations, Wiener-Hopf equations, singular integral equations); perturbation methods for partial differential equations; group theory and group representations. (Not offered 1975–76.)

H. Cheng

#### 18.308 Wave Motion (A)

Prereq.: 18.307

Year: G (2) 3-0-9

Review of linear waves. Nonlinear interactions, resonances, instabilities. Long waves, breaking, solitons. Korteweg-de Vries equation and the inverse scattering method. Nonlinear dispersive waves, multiphase modes. Random waves and closure. Nonlinear stability, transition phenomena. Special topics. (Not offered 1975–76.)

D. J. Benney

### 18.313 Probability

Prereq.: 18.002 or 18.02

Year: U (2) SD

3-0-9

Introduction to basic probabilistic techniques in all branches of science, with emphasis on stochastic processes. Topics selected from: Independent Random variables, renewal theory, elements of queueing theory, Markov processes, Brownian motion, the invariance principle, Poisson processes, branching processes. Content varies from year to year, and subject can be taken in successive years with the consent of the instructor.

### 18.314 Applied Combinatorial Analysis

Prereq.: 18.001 or 18.01

Year: U (2) 3-0-9

Applications of combinatorial methods to practical problem solving. Emphasis on problems involving discrete optimization. Techniques from graph theory, matching theory, network flows, linear and integer programming. Other topics include games, sorting and coding.

R. P. Stanley

### 18.315 Combinatorial Theory (A)

Prereq.: 18.002 or 18.02

Year: G (1) 3-0-9

Survey of combinatorial problems of current research interest. This year emphasis on asymptotic enumeration of combinatorial structures, extremal properties on partial orders and Boolean algebras, sums of vectors, constructions of degree sequence realizations. Content varies from year to year; may be repeated for credit.

D. J. Kleitman

## 18.318 Theory and Application of Finite Fields

Prereq.: 18.411 or 18.703

Year: U (2)

3-0-9

Structure of finite fields, with applications to coding theory and number theory. An introduction to equations over finite fields, including Gauss and Jacobi sums, reciprocity theorems, and special cases of the zeta function. Cyclotomy with applications to difference sets. Finite Fourier analysis and weight congruences for cyclic codes. A prior acquaintance with finite fields not needed. (Not offered 1975–76.)

R. P. Stanley

### 18.325 Topics in Applied Mathematics (A)

Prereq.: Permission of Instructor

Year: G (2)

3-0-9

Content varies from year to year. Second term 1976: readings from current research papers in Combinatorics. Topics to be chosen and presented by the class.

D. J. Kleitman

# 18.326 Numerical Methods of Applied Mathematics (A)

Prereq.: 18.075, 18.700

Year: G (2)

3-0-9

Introduction to the theory and application of numerical methods. Survey of numerical linear algebra, numerical solution of differential equations, approximation theory. Emphasis on finite difference, finite element, and spectral methods for solution of partial differential equations. Applications to equilibrium, diffusion, and wave problems, especially problems of current research interest. (Not offered 1975–76.)

### 18.350 Introduction to Fluid Mechanics

Prereq.: 18.075 or 18.301

Year: U (2) 3-0-9

Elementary theorems of ideal fluids. Some basic concepts drawn from the mathematical description. Potential flows; vortex motions. Waves; group velocity; normal modes; instability. Elements of the theory of viscous fluids; some exact solutions; slow flows; boundary layer theory.

D. J. Benney

### 18.356 Rotating Fluids (A)

Prereq.: 18.305, 18.350

Year: G (2)

3-0-9

General theory of rotating fluid motions; transient flows; effects of viscosity, stratification, and compressibility; non-linear interactions; wave motions; applications.

W. V. R. Malkus

#### 18.357 Seminar in Fluid Dynamics (A)

Prereq.: 18.305, 18.350

Year: G (1)

3-0-9

Discussion of current work on stability theory, non-linear flows, turbulence, hydromagnetics and geophysical fluid dynamics. Students to contribute a critique of a recent paper or the formulation and defense of a novel problem. W. V. R. Malkus

# 18.358 Hydrodynamic Stability and Turbulence (A)

Prereq.: 18.350 Year: G (2)

3-0-9

S. A. Orszag

Laminar flows, linear instability, finite-amplitude theory and turbulence. Examples in shearing flow, convection, rotating flow, and hydromagnetics. Compact reviews and demonstrations. Modified and singular perturbation theory developed for detailed flow studies. Statistical and optimal theories developed for turbulence studies. Applications to biophysics, engineering, geophysics and astrophysics. (Not offered 1975–76.)

# 18.370 Introduction to Dynamics in Astronomy

Prereq.: 18.075 or 18.301

Year: U (2)

3-0-9

Basic dynamical problems in astrophysics on all scales. Expanding universe, Newtonian and Einsteinian theories. Galaxies, mathematical description of stellar systems. Selected topics from galactic structure, interacting galaxies, star clusters. Interstellar medium, star formation, Emden spheres, stellar pulsations. Planetary and stellar orbits. (Not offered 1975–76.)

# 18:375 Stellar Dynamics and Galactic Structure (A)

Prereq.: 8.06; 18.076 or 18.302

Year: G (2) 3-0-9

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Types and compositions of galaxies; mass distributions and kinematics. General dynamics of stellar systems: Liouville and Boltzmann equations, isolating integrals; encounters, relaxation times; epicyclic stellar orbits. Dynamics of interstellar material: observations; hydromagnetic equations; gravitational stability of a gas. Theory of star clusters and elliptical galaxies. Gravitational stability of a disk-shaped galaxy; spiral structure. (Not offered 1975–76.)

### 18.376 Galaxies, and Their Dynamics (A)

Prereq.; 8.03; 18.076 or 18.301 or 18.370 Year: G (2)

3-0-9

Brief introduction to modern observational data including the Palomar Sky Survey, the Subble Atlas and Arp's Atlas of Peculiar Galaxies, followed by critical examination of existing dynamical theories on the origin and structures and groupings of galaxies. Ideas on galaxy formation. Dynamics of stars in elliptical galaxies, and of galaxies themselves in clusters: relaxation, dynamical friction, phase mixing, self-consistency, n-body computations. Stellar and continuum dynamics in disk galaxies: orbits, equilibria, instabilities, waves, modes, bars, spiral theories and experiments. Tidal interactions between palaxies. Some dynamics of galactic nuclei. Toomre

# 18.395 Mathematical Theory of Modern Physics (A)

Prereq.: 18.302 or 18.305 or 8.321

Year: G (1)

Emphasizes modern applications of group theory to particle physics. Representations of the Rotation, Lorentz, Poincaré and Symmetric Groups. Internal Symmetry Groups. Characters. Young Tableaux. Gauge Groups. Applications to the spectra and interactions of elementary particles, nuclei and atoms.

J. Mandula

### 18.397 General Relativity (A)

Prereq.: 18.965, 18.966, 18.969

Year: G (2) 3-0-9

Structure of space-time manifold; elements of tensor analysis and differential geometry, differential forms, spinors, topology. Symmetry groups. General covariance. Principle of equivalence. Einstein field equations. Exact solutions. Post-Newtonian approximation. Gravitational radiation and radiation reaction. Conservation laws and "pseudotensors. Classification of the Weyl tensor, equation of geodesic deviation, peeling theorem. Continuum mechanics. Thermodynamics. Geometrical optics. Space-like vector fields. Gravitational collapse, singularity theorems. Applications to astrophysics cosmology. Content varying from year to year. (Not offered 1975-76.) I. M. Singer

### 18.401 Control Theory

Prereq.: 18.03 or 18.031

Year: U (2)

3-0-9

Introduction to foundations of control theory. Controllability, observability, dynamic programming, maximum principle, optimal control, Lyapunov stability, and related topics discussed in suitable linear and nonlinear systems, with an emphasis on the geometrical and physical meaning of examples and proofs. *G. Carpenter* 

### 18.404 Seminar in Computer Calculus

Prereq.: 18.001 Year: U (1, 2)

2-2-2

A freshman-sophomore level subject designed to enhance the understanding of calculus concepts through formulation and solution of problems in calculus on computers, and to provide calculus students with a working knowledge of modern computer systems. Introduction to computer languages BASIC, MACSYMA. Numerical and symbolic computer techniques, differentiation, integration; iteration and perturbation procedures, etc. Students have access to time-shared computing systems. Enrollment limited. Term 1: R. LaBudde Term 2: P. S.-H Wang

### 18.411 Applied Algebra

Prereq.: — Year: U (2) 3-0-9

Applications of modern algebra, at an introductory level, to selected topics in discrete mathematics of interest to computer science and related fields. Modern and classical algebra developed as it is needed. Topics: coding theory, shift registers, finite Fourier transforms, Boolean algebra, theory of equations.

C. Greene

# 18.420 Introduction to the Mathematical Theory of Computation

Prereq.: —— Year: U (2) SD 3-0-9

Abstract models of computation: finite automata, Turing machines, recursive functions theory. Computable and non-computable functions. Grammars and string processing. Representation of programs and data.

M. Fredman

### 18.424 Seminar in Computation

Prereq.: —— Year: U (1, 2) 3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussion by students on topics taken from current journal or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite. (Not offered 1975–76.)

P. S.-H. Wang

#### 18.425 Theory of Computers (A)

Prereq.: Permission of the Instructor Year: G (2)

3-0-9

Study of various algorithmic problems from the point of view of their computational complexity. Methods for establishing lower bounds on complexity as well as elegant algorithms for achieving upper bounds presented. The computational problems treated are from algebra, logic, combinatorics, formal language theory and switching theory. Students should have had a basic algebra subject, including linear algebra. M. O. Rabin

### 18.427 Formal Language Theory (A)

Prereq.: 18.411 or 18.703

Year: G (2)

3-0-9

Methods of language specification. Finite-state, context-free, context-sensitive grammars and general rewriting systems. Deterministic and non-deterministic, acceptors: finite-state, pushdown store, stack machines and linear bounded automata. Closure properties of languages under various mappings. Abstract families of languages. Time and spacebounded Turing machines. Decidable and undecidable problems. Parsing of context-free languages. (Not offered 1975-76.) M. Fredman

### 18:428 Advanced Topics in Computation (A)

Prereg.: Permission of Instructor Year: G (1)

2-3-7

Project subject on the refinement and application of computer systems for non-numerical applied mathematics. Use of MACSYMA, a computer system for manipulation of algebraic expressions. Algorithmic and heuristic methods of mathematics. Projects may be solution of actual mathematical problems using MACSYMA. (Not offered 1975-76.) P. S.-H. Wang

### 18.437J Algorithms (A)

(18.429)

(Same subject as 6.851J)

Prereq.: 18.03 or 18.031

Year: G (1) 3-0-9

The design and mathematical analysis of basic algorithms for computer science. Parsing, sorting, searching, data structures, shortest path methods; computation of integer, matrix, polynomial, and string operations. Emphasis on techniques leading to asymptotically efficient methods. (Familiarity with computer programming required.) M. L. Fredman, M. J. Fischer

#### 18.438J Advanced Algorithms (A) (New)

(Same subject as 6'.854J)

Prereq.: 6.851J or 18.437J, 6.043 or

18,700 Year: G (2) 3-0-9

Exploration in greater depth of some topics from 6.851J/18.437J. New topics including network and matroid algorithms, advanced graph algorithms, and problems from logic. Greater emphasis on the underlying models of computation and on precise mathematical analysis.

M. L. Fredman, M. J. Fischer

### 18.439 Decidability (New)

Prereq.: 18.511 or 18.515

Year: G (1) 3-0-9

Comprehensive treatment of modern decidability results for algebraic systems, second-order theories, various theories of ordered sets etc. Methods used include automata theory and algebraic tools. Lower bounds on the complexity of the decision problems in question also given. Students should be familiar with the algebraic theory of fields.

M. O. Rabin

### 18.440 Probability and Random Variables

(Revised: Unit change) Prereq.: 18.002 or 18.02

Year: U (1, 2) SD

4-0-8

A standardized introduction to probability covering the basic topics useful in applications. Probability spaces, random variables, distribution functions, expected value. Binomial, geometric, hypergeometric, Poisson distributions. Uniform, exponential. normal, gamma and beta distributions. Mean, variance, moments and generating functions. Conditional probability, Bayes theorem, joint distributions, and distributions of transformed random variables. Tchebychev inequality, law of large numbers and central limit theorem. Multivariate normal distribution, covariances and correlation. Random walk and Markov processes. Some applications to statistics and decision theory.

Term 1: H. Chemoff Term 2: Staff

(A, except XVIII) (18.304)

(Revised: Unit change)

18.441 Statistical Inference

Prereq.: 6.041 or 18.440 or 18.313

Year: U (1, 2), G (1, 2)

4-0-8

Introduction to statistical inference. Decision theory, hypothesis testing, point and interval estimation. Bayesian methods, maximum-likelihood and likelihood-ratio tests. Chi-square goodness of fit tests. Comparison of populations by parametric and non-parametric methods. Analysis of variance regression and correlation. Sequential analysis if time permits. Treatment more mathematical than that of 18.05 and more detailed in its treatment of statistics.

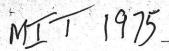
Term 1: Staff Term 2: H. Chernoff

#### 18.443 Statistics for Applications

Prereq.: 18.440 or 18.313 or 6.041 Year: U (1) SD

4-0-8

A broad treatment of statistics concentrating on specific statistical techniques used in science and industry. Topics: hypothesis testing and estimation. Chi-square goodness of fit, regression, correlation, analysis of variance and experimental design. Treatment more oriented toward application and less toward theory than 18.441. Staff



Logic

### 18.445J Introduction to Stochastic Processes (A)

(18.345)

(Same subject as 15.073J)

Prereq.: 18.313 or 18.440 or 6.041

Year: G (1, 2)

4-0-8

Introduction to the theory and application of stochastic processes. Examples of empirical phenomena for which stochastic processes provide mathematical models. Random walk, gambler's ruin, recurrent events, discrete-time Markov chains, Branching processes, Poisson processes, non-homogeneous Poisson processes, compound Poisson processes, renewal theory. Continuous-time Markov chains, birth and death processes, queueing theory and Brownian motion. Time permitting, additional topics chosen from: semi-Markov processes, Markov renewal processes, Markov decision process, inventory theory and Gaussian processes.

Term 1: A. J. Petkau Term 2: A. Barnett

### 18.455 Analysis of Variance and Design of Experiments (A) (New)

Prereq.: 18.700, 18.441

Year: G (1)

4-0-8

Theory and application of techniques used in analysis of variance and design of experiments. General linear hypothesis. Regression. Various models in analysis of variance and the analysis of covariance. Simultaneous confidence intervals. Design of randomized blocks and latin squares. Factorial experiments and confounding. Response surface methodology. A. J. Petkau

### 18.456 Multivariate Methods in Statistics (A) (New)

Prereq.: 18.700, 18.441

Year: G (2)

4-0-8

Theory and application of commonly used techniques involving multivariate data. Attention devoted to specific applications, and to computational facilities for applying the methods. Topics selected from the following: multivariate regression, discriminant analysis and pattern classification using nearest neighbor methods. Cluster analysis, factor analysis and principal components. Multidimensional scale analysis. Contingency tables.

H. Chernoff

### 18.457J Statistics for Model Building (A) (New)

(Same subject as 15.063J)

Prereq.: 18.441 or 18.443 or 15.075

Year: G (2) 3-0-9

Designed to be a second subject in data analysis and applied statistics for those interested in model building, calibration and validation. Topics: linear and non-linear regression, analysis of variance, robust and Bayesian regression, selection of variables, forecasting, statistical computing and graphical presentation of data. Term paper required. R. E. Welsch, H. Chernoff

# 18.459 Statistical Laboratory (A)

Prereq.: Permission of Instructor

Year: G (1, 2)

2-0-4

Lectures consist of presentations of applied problems with or without solutions. Students expected to give a presentation based on the literature or the solution of a minor problem. The solution of a major problem serves as the basis for a Master's thesis. Staff

### 18.465 Topics in Statistics (A)

(18.349)

Prereq.: Permission of Instructor

Year: G (1, 2)

3-0-9

Introduction to some special theoretical topics in mathematical statistics at an intermediate level. Students assumed to be familiar with the elements of probability theory and the fundamental concepts and basic techniques of statistical inference. Topics chosen from: optimal design of experiments, large sample theory, sequential analysis and multivariate analysis. Other topics chosen in accordance with the interests of the students. (Offered only second term 1975-76.) A. J. Petkau

### 18.466 Mathematical Statistics (A)

(18.185)

Prereq.: 18.125, 18.313

Year: G (1) 3-0-9

Sampling, tests of significance, power of tests, estimation, confidence regions, order statistics, directional data (circles and spheres), analysis of variance, decision theory. More mathematically advanced than 18.441; uses measure theory and integration over general probability spaces. R. M. Dudley

### 18.511 Introduction to Mathematical Logic

Prereq.: Year: U (2) 3-0-9

Prepositional and predicate logic. Elementary model theory, completeness, compactness, and Lowenheim-Skolem theorems; elementary recursion theory, enumeration and recursion theorems; elementary set theory, ordinals, cardinals, and transfinite induction. Godel incompleteness theorems. Special additional topics as time permits. (While this subject has no formal prerequisite, any Course XVIII subject with first decimal digit one or higher

P. Lavori

### 18.515 Mathematical Logic (A)

will be adequate preparation.)

Prereq.: Permission of Instructor Year: G (1) 3-0-9

Similar to 18.511 but at a faster pace. Additional topics varying from year to year, for example: model-theoretic preservation theorems, elementary priority arguments, or constructible sets. (While this subject has no formal prerequisite, any Course XVIII subject with first decimal digit one or higher will be adequate preparation.) G.-C. Rota

### 18.525 Seminar in Logic (A, except XVIII)

Prereq.: 18.515 or 18.551 or 24.241

Year: G (2)

Directed presentation by students of topics of interest from various areas of logic. Sources include current journals as well as texts and monographs. Content varies from year to year; may be repeated for credit. E. M. Kleinberg

### 18.565 Recursion Theory (A)

Prereq.: 18.511 or 18.515

Year: G (2)

3-0-9

Review of elementary recursive function theory. Ordinary recursion theory. Degree theory. Priority arguments. E. M. Kleinberg

### 18.566 Higher Recursion Theory (A)

Prereq.: 18.565 Year: G (2)

3-0-9

Recursion on initial segments of ordinals and in objects of finite type. Post's problem, hyperregularity, stability, k-sections. (Not offered 1975-76.)

G. E. Sacks

### Algebra and Number Theory

### 18.575 Theory of Models (A)

Prereq.: 18.511 or 18.515

Year: G (1) 3-0-9

The compactness theorem, elementary substructures, indiscernibles, Morley rank, categoricity in power. Model completeness, elimination of quantifiers. Ultraproducts. P. Lavori

### 18.576 Theory of Models (A)

Prereq.: 18.575 Year: G (2) 3-0-9

Applications to algebraic theories: Hilbert's 17th problem; Ax and Kochen's work on p-adic fields; algebraically closed division rings and groups; model-completeness or categoricity of various algebraic theories. (Not offered 1975-76.)

### 18.585 Set Theory (A)

Prereq.: 18.511 or 18.515

Year: G (1) 3-0-9

Axiomatic set theory. Constructible sets. Independence of the axiom of choice and of the generalized continuum hypothesis. A model in which every set of reals is Lebesgue measurable. Measurable cardinals and the existence of nonconstructible sets of integers. N. S. Kroonenberg

### 18.586 Set Theory (A)

Prereq.: 18.585 Year: G (2) 3-0-9

Boolean-valued models. Independence of Souslin's hypothesis. Admissible ordinals. Large cardinal axioms. The axiom of determinateness. (Not offered 1975-76.) E. M. Kleinberg

### 18.587 Infinitary Combinatorics (A)

Prereq.: 18.585 Year: G (2) 3-0-9

Advanced treatment of pure set theory with an emphasis on problems of current research interest. New set theoretic axioms inspired by generalizations of Ramsey's theorem. Connections with indiscernibility and strengthened Lowenhein-Skolem theorems. Related equiconsistency results. Measurable cardinals and compact cardinals. Relative strengths of the various partition relations and large cardinal axioms in general. Infinite exponent partition relations and connections with the axiom of determinateness. (Not offered 1975-76.) E. M. Kleinberg

### 18.595 Seminar on Current Topics in Logic (A)

Prereq.: 18.565, 18.575, 18.585

Year: G (1, 2)

Arr.

Analysis of results of current interest in logic. Students present for general discussion recent developments in the field. Formal and informal sources used. Topics vary from year to year; may be repeated for credit.

G. E. Sacks

### 18.700 Linear Algebra

Prereq.: 18.002 or 18.02 Year: U (1, 2) SD

3-0-9

Basic subject on matrix theory and linear algebra, emphasizing the topics most useful in other disciplines. Covers the fundamental theory: systems of equations, vector spaces, determinants, eigenvalues, similarity, positive definite matrices and quadratic forms Applications given to such topics as Gauss elimination with pivoting, least squares approximations, stability of systems of differential equations, and Rayleigh-Ritz variational techniques. Compared with 18.710 this subject is oriented more towards linear algebra as used in applications, and less towards theorems and proofs. W. G. Strang and Staff

### 18.701 Algebra I

Prereq.: 18.002 or 18.02 Year: U (1)

3-0-9

#### 18.702 Algebra II

Prereq.: 18.701 Year: U (2) 3-0-9

Similar to the 18.700-18.703 sequence, but somewhat more extensive and theoretical. Some experience with linear equations and matrices (as in 18.03) helpful. First term: determinants, matrix groups, eigenvectors, symmetry, Sylow theorems, bilinear forms. Second term: ideals, polynomial rings, unique factorization, modules, Jordan form for matrices, extension fields and Galois theory

Term 1: R. D. Shafer Term 2: M. Artin

#### 18.703 Modern Algebra

Prereq.: 18.002 or 18.02 Year: U (1, 2) SD

3-0-9

A one term treatment, covering the traditional algebra topics which have found greatest application in science and engineering as well as in other mathematical disciplines: group theory, with emphasis on finite groups; ring theory, including ideals, unique factorization. in polynomial and Euclidean rings; field theory, including properties and applications of finite fields. 18.700 and 18.703 together cover most of basic algebra. 18.700 or 18.710 should precede 18.703 if both subjects are to be taken. Open to students with some experience in abstract thinking (college level mathematics).

Term 1: S. L. Kleiman Term 2: D. G. Quillen

## 18,704 Seminar in Algebra and Number

Prereq.: 18.702 or 18.703

Year: U (1, 2)

3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussion by students on topics taken from current journals or from texts not regularly used in other mathematics subjects.

Term 1: S. L. Kleiman

Term 2: ---

### 18.705 Algebra III (A)

Prereq.: 18.701, 18.702; or 18.700,

18.703 Year: G (1) 3-0-9

Commutative algebra. Inseparable and transcendental extensions of fields; noetherian rings and modules, basis theorem, primary decomposition; localization and tensor product; integral dependence, normalization, nullstellensatz, classical ideal theory; dimension theory; introduction to homological algebra.

A. P. Mattuck

#### 18.706 Algebra IV (A)

Prereq.: 18.702 or 18.700, 18.703

Year: G (2) 3-0-9

Basic topics in non-commutative algebra, to be selected from such areas as: Wedderburn structure theory of rings, group representations, Lie and other non-associative algebras, algebraic groups. Content varies from year to year.

B. Kostant

18.710 Abstract Linear Algebra

Prereg.: 18.002 or 18.02

Year: U (1, 2) SD

3-0-9

An algebraic treatment of linear algebra including vector spaces, systems of linear equations, bases, linear independence, matrices, determinants, eigenvalues, inner products, quadratic forms, and canonical forms of matrices. More theoretical than 18.700, with less emphasis on matrix theory and applications.

Term 1: D. G. Quillen Term 2: B. Kostant

#### 18.715 Homological Algebra (A)

Prereq.: 18.705 Year: G (1) 3-0-9

### 18.716 Homological Algebra (A)

Prereq.: 18.715 Year: G (2) 3-0-9

Classifying spaces of categories, higher K-groups, exact sequences in K-theory, calculations. Familiarity with homotopy and cohomology groups required. (Not offered 1975–76.)

D. G. Quillen

18.725 Algebraic Geometry (A)

Prereq.: 18.705 Year: G (1) 3-0-9

Introduction to contemporary algebraic geometry. Affine and projective schemes. Basic computations of cohomology groups of quasi-coherent sheaves. Riemann-Roch theorem for curves and surfaces.

M. Artin

### 18.727 Topics in Algebraic Geometry (A)

Prereq.: 18.725 Year: G (2) 3-0-9

Topics varying from year to year; may be repeated for credit. Topics in the past few years have included generalized Riemann-Roch theorem, etale homology, algebraic spaces, Riemann hypothesis for curves over finite fields, deformation theory, topics in the theory of algebraic surfaces. 1975–76: Continuation and completion of 18.725.

M. Artin

### 18.735 Topics in Algebra (A)

Prereq.: 18.702 or 18.703

Year: G (1) 3-0-9

Content varies yearly; may be repeated for credit. Topics in the past few years have included algebraic groups, non-commutative rings and algebras, Lie algebras, non-associative algebras, local rings, algebraic K-theory, arithmetic theory of quadratic forms. (Not offered 1975–76.)

N. Ankeny

### 18.737 Linear Algebraic Groups (A)

Prereq.: 18.705 Year: G (2) 3-0-9

An introduction to the classification of affine groups over an algebraically closed field via their representations as groups of invertible matrices. Topics: the representation theorem, semi-invariants, algebraic tori, the Jordan decomposition, the Borel fixed-point theorem, the Lie-Kolchen theorem, Borel subgroups, Cartan subgroups, the Weyl group, root systems, Bruhat decomposition, Dynkin diagrams, Young diagrams. No algebraic geometry presupposed. (Not offered 1975–76.)

### 18.755 Lie Groups and Lie Algebras (A)

Prereq.: 18.101 Year: G (1) 3-0-9

Topology and structure of the classical linear groups. Basic theory of Lie groups: exponential map, subgroups, homogeneous spaces, automorphism groups, Campbell-Hausdorff formula. Structure of Lie algebras. S. Helgason

### 18.756 Analysis of Lie Groups (A)

Prereq.: 18.755 Year: G (2) 3-0-9

Semisimple Lie groups and symmetric spaces. Topics in function theory on symmetric spaces, such as Fourier analysis and Radon transform, invariant differential operators and potential theory. Emphasis on connections with classical analysis and representation theory.

S. Helgason

#### 18.757 Representations of Lie Groups (A)

Prereq.: 18.755 Year: G (1) 3-0-9

Unitary representation of Lie groups, in the framework of Hamiltonian mechanics and quantizations.

B. Kostant

### Topology and Geometry

#### 18.775 Algebraic Number Theory (A)

Prereq.: 18.705 Year: G (1) 3-0-9

#### 18,776 Algebraic Number Theory (A)

Prereq.: 18.775 Year: G (2) 3-0-9

Study of algebraic number fields. Ideals, class-numbers, units, zeta functions, prime ideal theorem, abelian L-series and reciprocity laws. Non-abelian L-series, Chebotarev density theorem, Brauer-Siegel theorem and class-number problems.

H. M. Stark

### 18.781 Theory of Numbers

Prereq.: 18.100; 18.700 or 18.701 or 18.703

78.703 Year: U (1) 3-0-9

Study of primes, congruences, and arithmetic functions and proofs of their asymptotic formulae. Approximations of the real numbers by rationals, Kronecker's theorem, and the introduction of geometry of numbers. Quadratic forms and quadratic number fields. Elementary proof of the prime number theorem. (Alternate years. Offered 1975–76.) J. Friedlander

#### 18.785 Analytic Number Theory

Prereq.: 18.115 Year: G (2) 3-0-9

Discussion of Dirichlet series including the Riemann zeta function and Dirichlet L-functions. Proof of the prime number theorem and generalizations to primes in arithmetic progressions. Sign change problems including the logarithmic integral of x minus the number of primes ≤ x. Asymptotic formula for the partition functions. (Not offered 1975–76.)

H. M. Stark

#### 18,786 Topics in Number Theory

Prereq.: Permission of Instructor Year: G (1, 2) 3-0-9

Topics varying from year to year; may be repeated for credit. Topics in the past few years included: Diophantine analysis and transcendence: Thue-Siegel-Roth theorem, Gelfond-Schneider and Baker theorems; quadratic number fields and complex multiplication. (Not offered 1975–76.)

H. M. Stark

# 18.794 Seminar in Algebra and Number Theory

Prereq.: —— Year: U (1, 2) 3-0-9

Seminars in several topics for mathematics majors. Each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussions by students on topics taken from current journals or from texts not regularly used in other mathematics subjects. (Not offered 1975–76.) Staff

# 18.901 Introduction to Topology (A except XVIII)

Prereq.: 18.100; 18.701 or 18.703

Year: U (1, 2) G (1, 2) 3-0-9

Topological spaces, connectedness, compactness, continuous functions, separation axioms, product spaces, function spaces, metrization theorems. The fundamental group, A one-term introduction covering topics fundamental to modern analysis and geometry. Intended for those going on for graduate work.

Term 1: J. R. Munkres Term 2: M. W. Davis

### 18.903 Introduction to Algebraic Topology

Prereq.: 18.100; 18.701 or 18.703

Year: U (1) 3-0-9

Introduction to some of the ideas of algebraic topology, but less advanced than 18.901 and 18.905. The notions of homotopy, zero and first cohomology groups, and their properties studied. Proof of the Jordan curve theorem. Classification of two-dimensional manifolds. (Not offered 1975–76.)

F. P. Peterson

### 18.904 Seminar in Topology

Prereq.: 18.901 Year: U (1) 3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Reports and discussion by students on topics taken from current journals or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite.

F. P. Peterson

#### 18.905 Algebraic Topology (A)

Prereq.: 18.702 or 18.700, 18.703; 18.901 Year: G (1)

3-0-9

### 18.906 Algebraic Topology (A)

Prereq.: 18.905 Year: G (2) 3-0-9

Fundamental group, covering spaces, simplicial homology, simplicial approximation manifolds. Homology and cohomology of topological spaces, universal coefficient theorem, plus additional topics to be chosen by the instructor.

E. Miller

### 18.915 Graduate Topology Seminar (A)

Prereq .: 18.906 Year: G (1) 3-0-21

Study and discussion of important original papers in the various parts of algebraic and differential topology.

D. M. Kan

### 18.917 Advanced Topology (A)

Prereq.: 18.906 Year: G (1) 3-0-9

Content varying from term to term so that graduate students taking the subject in successive terms may have an introduction to several important phases of topology such as nomotopy theory, cohomology theory, fibre spaces, K-theory, combinatorial topology, and/or differential topology, 1975-76: Homotopy theory. G. W. Whitehead

#### 18.925 Differential Topology (A

Prereq.: 18.906 Year: G (2) 3-0-9

Manifolds, vector bundles, characteristic classes, cobordism, smoothing theory. J. R. Munkres

#### 18.950 Elementary Differential Geometry

Prereq.: 18.100 Year: U (2) 3-0-9

Emphasis on classical theory of space curves and surfaces. Frenet formulas, fundamental forms, theorems of Meusnier and Euler, lines of curvature, asymptotic and conjugate lines, geodesics, equations of Gauss, Cadazzi and Weingarten, special surfaces, mapping problems and existence theorems. Additional topics to be chosen from geometry in the large, classical tensor theory and Riemannian geometry. H. Donnelly

#### 18.961 Elementary Differential Topology

Prereq.: 18.101, 18.901 Year: U (1)

3-0-9

Differentiable manifolds in Rn; differentiable mappings; transversality; Sard's theorem; intersection theory; the Euler characteristic of a manifold and the Lefschetz fixed point theorem; integration on manifolds, Stokes' heorem, DeRham cohomology. V. W. Guillemin

### 18.965 Geometry of Manifolds (A)

Prereg.: 18.101 Year: G (1) 3-0-9

### 18.966 Geometry of Manifolds (A)

Prereq.: 18.965 Year: G (2) 3-0-9

Differentiable manifolds and mappings, triangulations and deRham cohomology, geometric integration theory.

18.965: H. Hecht

18.966: Not offered 1975-76.

### 18.969 Topics in Geometry (A)

Prereq.: 18.966 Year: G (1) 3-0-9

Advanced topics in the geometry of manifolds. Curvature and Characteristic classes, refined characteristic classes (Chern-Simons), foliations. Continued in second term as 18.397. (Not offered 1975-76.) I. M. Singer

#### 18.975 Elliptic Operators (A)

Prereq.: 18.101, 18.905 Year: G (1)

3-0-9

### 18.976 Elliptic Operators (A)

Prereq.: 18.975 Year: G (2) 3-0-9

Differential, pseudodifferential and Fourier integral operators on manifolds; applications in the theory of elliptic operators, and in particular index theorems and asymptotic behavior of eigenvalues. (Not offered 1975-76:) V. W. Guillemin

#### 18.994 Seminar in Geometry

Prereq.: Year: U (1, 2) 3-0-9

Seminars in several topics, for math majors. Each under direction of a faculty member whose interest is in the field of the seminar. Reports and discussions by students on topics from current journals or texts not regularly used in other math subjects. (Not offered 1975-76.) Staff

### 18.999 Mathematical Reading

Prereq .: -Year: G (1, 2) Arr.

Reading of advanced mathematical treatises under supervision of a member of the Department. For graduate students desiring advanced work not provided in regular subjects. M Artin