MIT

1985

Department of Mathematics

(Course 18)

Undergraduate Study

M

Professors Emeriti

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Bachelor of Science in Mathematics Course XVIII

An undergraduate degree in mathematics provides an excellent basis for graduate work in mathematics or computer science, or for employment in such mathematics-related fields as systems analysis, operations research, or actuarial science.

Because the career objectives of undergraduate mathematics majors are so diverse, each undergraduate's program is individually arranged through collaboration between the student and his or her faculty advisor. In general, students are encouraged to explore the various branches of mathematics, both pure and applied. Considerable elective time is available in each of the mathematics programs.

Undergraduates wishing to work in small groups under the supervision of a faculty member may elect to participate in a mathematics seminar. This is normally done during the junior year or the first semester of the senior year. The experience gained from active participation in a seminar conducted by a research mathematician is particularly valuable for a student planning to pursue graduate work in some branch of mathematics or a related field.

There are three undergraduate programs that lead to the degree Bachelor of Science in Mathematics: a General Mathematics Option, an Applied Mathematics Option for those who wish to specialize in that aspect of mathematics, and a Theoretical Mathematics Option for those who expect to pursue graduate work in pure mathematics.

The interaction that takes place in the classroom is an important component of the learning experience in mathematics. Therefore, in none of the programs is advanced standing credit accepted toward fulfillment of any part of the Departmental Requirements other than 18.03. It is accepted, however, as Unrestricted Elective.

Bachelor of Science in Mathematics Course XVIII General Mathematics Option

This option is the one followed by most students who major in mathematics. Besides the General Institute Requirements, the requirements consist of 18.03 Differential Equations and eight 12-unit subjects in Course 18 of essentially different content, including at least six advanced subjects (first decimal digit one or higher). This leaves available 84 units of unrestricted electives. The requirements are flexible in order to accommodate several categories of students:

1) those who wish to pursue programs that combine mathematics with a related field (such as computer science, physics, economics, or management), 2) those who wish to divide their time between theoretical and applied mathematics, and 3) those who wish to use mathematics as a general Institute major.

Applied mathematics is the mathematical study of general scientific concepts, principles, and phenomena which, because of their widespread occurrence and application, relate or unify various disciplines. The core of the program at MIT concerns the following principles and their mathematical formulations: propagation, equilibrium, stability, optimization, cybernetics, statistics, and random processes. The undergraduate program provides a general introduction to most areas of applied mathematics and to several specific areas for study in greater depth.

Freshmen interested in applied mathematics should consider taking 18.001 and 18.002 Calculus followed as soon as possible by 18.03 Differential Equations.

Sophomores interested in applied mathematics typically survey the field of applied mathematics by enrolling in both 18.310 and 18.311 Principles of Applied Mathematics. Subject 18.310, given only in the first term, is devoted to the discrete aspects of the subject and may be taken concurrently with 18.03. Subject 18.311, given only in the second term, is devoted to continuous aspects and makes considerable use of differential equations.

The subjects in Group I of the program correspond roughly to those areas of applied mathematics making heavy use of discrete mathematics, while Group II emphasizes those subjects which deal mainly with continuous processes. Naturally, there is a good deal of overlap; for example, such subjects as probability or numerical analysis have both discrete and continuous aspects. In general, students in the applied mathematics option are encouraged to acquire as good a background as possible in both types of applied mathematics.

For those who wish to emphasize particular areas within the applied mathematics curriculum, sample programs are available in the Undergraduate Mathematics Office for the following specialties: combinatorics, computer science, fluid dynamics, numerical analysis, statistics, and theoretical physics.

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Bachelor of Science in Mathematics
Course XVIII
Applied Mathematics Option

General Institute Requirements	Total Units
Science Requirement	60
Humanities, Arts, and Social Sciences Requirement	72
The Science Distribution Requirement can be satisfied by 18.03 in the Departmental Program, plus appropriate subjects totaling	24
Laboratory Requirement	12

Departmental Program

Subject names below are followed by credit units, by prerequisites if any (corequisites in italics)

Required Subjects:		60
18.03	Differential Equations, 12; 18.02	
18.310	Principles of Applied Mathematics, 12; 18.02	
18.311	Principles of Applied Mathematics, 12; 18.03	
One of the following two subjects:		
18.04	Complex Variables with Applications, 12; 18.03	
18.284	Introduction to Functions of a Complex Variable, 12; 18.03	
One of the following two subjects:		
18.411	Applied Algebra, 12; 18.06 or 18.710, 18.063 or 18.703	
18.06	Linear Algebra, 12; 18.02	
Restricted Electives:		48
At least four subjects from the following two groups with at least one subject from each group		
Group I		
18.440	Probability and Random Variables, 12; 18.02 *or*	
18.313	Probability, 12; 18.02	
18.441	Statistical Inference, 12; 18.440 or 18.313* *or*	
18.443	Statistics for Applications, 12; 18.440 or 18.313*	
18.314	Applied Combinatorial Analysis, 12; 18.01	
18.420J	Automata, Computability, and Complexity, 12; 18.310*	
Group II		
18.330	Introduction to Numerical Analysis, 12; 18.03	
18.301	Introduction to Physical Mathematics I, 12; 18.03	
18.302	Introduction to Physical Mathematics II, 12; 18.301, 18.04 or 18.284	
18.354	Fluid Mechanics, 12; 18.04 or 18.302*	
Unrestricted Electives		84
Total Units Required for the S.B. Degree		360

* Alternate prerequisites are also listed in the subject description.

Theoretical mathematics (or "pure" mathematics) is the study of the basic concepts and structures that underlie the mathematical tools used in science and engineering. Its purpose is to search for a deeper understanding and an expanded knowledge of mathematics itself.

Traditionally, pure mathematics has been classified into three general fields: analysis, which deals with continuous aspects of mathematics; algebra, which deals with discrete aspects; and geometry. The undergraduate program is designed so that students become familiar with each of these areas. Students may also wish to explore such other topics as logic, number theory, complex analysis, and subjects within applied mathematics.

The subject 18.100B Analysis I is basic to the program. Since this subject is strongly proof-oriented, many students find an intermediate subject such as 18.06 Linear Algebra or 18.710 Abstract Linear Algebra useful as preparation.

Some flexibility is allowed in this program. For instance, students may, with permission, substitute 18.100A for 18.100B; and they may substitute 18.710 Abstract Linear Algebra, plus 18.703 Modern Algebra, for the recommended algebra sequence 18.701-18.702. Similarly, a thesis or (less desirably) a first-year graduate subject may be substituted for the Seminar.

Bachelor of Science in Mathematics
Course XVIII
Theoretical Mathematics Option

General Institute Requirements	Total Units
Science Requirement	60
Humanities, Arts, and Social Sciences Requirement	72
The Science Distribution Requirement can be satisfied by 18.03, in the Departmental Program, plus appropriate subjects totaling	24
Laboratory Requirement	12

Departmental Program

Subject names below are followed by credit units, and by prerequisites if any (corequisites in italics)

Required Subjects:		72
18.03	Differential Equations, 12; 18.02	
18.100B	Analysis I, 12; 18.03*	
One of the following three subjects:		
18.101	Analysis II, 12; 18.100B, 18.701*	
18.102	Analysis II, 12; 18.100B	
18.103	Fourier Analysis -- Theory and Applications, 12; 18.100B	
	plus	
18.701	Algebra I, 12; 18.02	
18.702	Algebra II, 12; 18.701	
18.901	Introduction to Topology I, 12; 18.100B	
Restricted Electives:		36
An upper level Mathematics Seminar[1] (12 units)		
Two additional Course 18 subjects of essentially different content, with the first decimal digit one or higher (24 units)		
Unrestricted Electives		84
Total Units Required for the S.B. Degree		360

Inquiries

Inquiries regarding academic programs may be addressed to Joanne Murray, Undergraduate Mathematics Office, Room 2-108, MIT, Cambridge, Massachusetts 02139, (617) 253-2416.

The following information sheets are available in Room 2-108: Undergraduate Subjects in Mathematics; Careers in Mathematics; Thinking of Majoring in Mathematics?; and Applied Mathematics: Sample Programs.

* Alternate prerequisites are also listed in the subject description.

1 These seminars are 18.104, 18.504, 18.704, 18.904, and 18.994.

Mathematics

General Mathematics

18.UR Undergraduate Research

Prereq.: —
U (1, 2)
Arr.

Undergraduate research opportunities in Mathematics. Permission required in advance to register for this subject. For further information consult the Departmental Coordinator.
D. A. Vogan, Staff

18.001 Calculus

Prereq.: Assumes some prior knowledge of calculus
U (1) **Next offered 1986-87**
5-0-7

Emphasizes the concepts and techniques of calculus relevant to science and technology. Mathematical formulation of problems and approximate methods of solution. Substantial review of limits and differentiation with discussion of some advanced methods. Intensive study of integration; perturbation and iteration procedures; stability; series; summation techniques; asymptotics; numerical analysis; vector algebra; special topics.
H. P. Greenspan

18.002 Calculus

Prereq.: 18.001 or 18.01 or 18.011 or 18.012
U (1, 2) **Next offered 1986-87**
5-0-7

Continues 18.001. Presents the concepts and techniques of calculus relevant to science. Vector algebra, analytic geometry, planetary motion, orbit stability, partial differentiation, functions of several variables, Taylor series, extremal problems, linear programming examples, numerical methods, multiple integrals, approximate and asymptotic methods of evaluation, applications; vector calculus, gradient, curl, theorems of Stokes, Green & Gauss, conservation laws, fluid motion.
D. J. Benney

18.01 Calculus

Prereq.: —
U (1, 2)
5-0-7

Differentiation and integration of functions of one variable, with applications. Concept of function, limits, and continuity. Differentiation rules, application to graphing, rates, approximations, and extremum problems. Mean-value theorem. Definite and indefinite integration. Fundamental theorem of calculus. Applications of integration to geometry and science. Elementary functions. Techniques of integration. Approximation of definite integrals, improper integrals, l'Hôpital's rule, and Taylor series.
Term 1: *D. A. Vogan*
Term 2: Information: D. A. Vogan.

18.011 Calculus

Prereq.: Assumes some prior knowledge of calculus
U (1)
5-0-7

Reviews 18.01 material in five weeks. Infinite series. Taylor's formula. Probability. Vectors, vector-valued functions of one variable, space motion. Scalar functions of several variables, partial differentiation, gradient, approximation techniques.
A. P. Mattuck

18.012 Calculus with Theory

Prereq.: —
U (1)
5-0-7

Covers much the same material as 18.01, but at a deeper and more rigorous level. Emphasizes careful reasoning and understanding of proofs. Assumes knowledge of elementary calculus. Topics: axioms for the real numbers; the Riemann integral; limits, theorems on continuous functions; derivatives of functions of one variable; the fundamental theorems of calculus; Taylor's theorem; infinite series, power series, rigorous treatment of the elementary functions. Information: D. A. Vogan.

18.02 Calculus

Prereq.: 18.001 or 18.01 or 18.011 or 18.012
U (1, 2, S)
5-0-7

Calculus of several variables. Vector algebra in 3-space, determinants, matrices. Vector-valued functions of one variable, space motion. Scalar functions of several variables: partial differentiation, gradient, approximation techniques. Multiple integrals with applications. Vector fields, line and surface integrals, exact differentials, Green's theorem, Divergence Theorem, Stokes' Theorem. Additional topics: linear algebra (term 1); infinite series (term 2).
Term 1: *H. Rogers, Jr.*
Term 2: *S. Helgason*

18.021 Calculus

Prereq.: 18.011
U (2)
5-0-7

Continues 18.011: calculus of several variables, with elementary linear algebra and applications. Vector fields, line integrals, exact differentials. Elementary linear algebra, Jacobians. Green's Theorem, surface integrals, Divergence Theorem, Stokes' Theorem. Permission of instructor required for those not having 18.011.
D. S. Jerison

18.022 Calculus with Theory

Prereq.: 18.012
U (2)
5-0-7

Continues 18.012. Parallel to 18.02, but at a deeper level, emphasizing careful reasoning and understanding of proofs. Considerable emphasis on linear algebra and vector integral calculus.
D. A. Vogan

18.03 Differential Equations

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (1, 2, S) SD
4-0-8

Examples of initial value problems in science and engineering associated with single equations and with systems of first order equations. Methods of solution include graphical constructions, series, Laplace transforms, matrices, numerical integration and the phase plane. Emphasizes formulation of natural phenomena in terms of differential equations and interpretation of the solutions.
Term 1: *F. Morgan, Staff*
Term 2: *A. P. Mattuck, Staff*

18.04 Complex Variables with Applications

Prereq.: 18.03
U (1, 2, S)
4-0-8

Complex algebra and functions; analyticity; contour integration, Cauchy's theorem; singularities, Taylor and Laurent series; residues, evaluation of integrals; multivalued functions, potential theory in two dimensions; Fourier analysis and Laplace transforms. 18.04 and 18.075 may not both be taken for credit.
Term 1: *A. Toomre*
Term 2: *L. N. Trefethen*

18.05 Introduction to Probability and Statistics

Prereq.: 18.001 or 18.01 or 18.011 or 18.012
U (1, 2) SD
3-0-9

Elementary introduction, with applications to the social, physical, and life sciences. Relative frequency. Probability models. Combinatorics. Binomial and Poisson experiments. Normal approximation. Descriptive level of significance. Composite models. Chi-square approximation. Contingency tables. Hypothesis testing. Confidence regions. Random variables. Non-parametric methods. Normal distribution methods. Regression. Elements of decision theory. Information: H. Chernoff.

18.057 Computer Data Analysis Laboratory (A except XVIII)

Prereq.: 18.05 or 15.061
U (2) G (2)
3-2-7

Methods of data analysis using computers and microcomputers. Introduction to and practice with the most frequently encountered statistical software systems including MINITAB, SAS, and BMDP. Statistical analyses using Project Athena microcomputers. Emphasizes intuitive bases of common statistical procedures and the use of interactive computer packages to analyze a wide variety of data sets. No previous programming experience necessary. Some statistics background necessary.
D. Riceman

18.06 Linear Algebra

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (1, 2, S) SD
3-0-9

Basic subject on matrix theory and linear algebra, emphasizing topics useful in other disciplines: systems of equations, vector spaces, determinants, eigenvalues, similiarity, postive definite matrices. Applications to Gauss elimination with pivoting, least squares approximations, stability of differential equations, linear programming, and game theory. Compared with 18.710, more emphasis on matrix calculations and applications. Information: G. Strang.

18.063 Introduction to Algebraic Systems

Prereq.: 18.02 or 18.002 or 18.021 or 18.022
U (1) SD
4-0-8

Introduction to algebraic systems primarily for students interested in computer and information sciences, with emphasis on finite systems. Reviews elementary set theory, natural numbers, modular arithmetic, induction, counting arguments. Elementary number theory and group theory. Applications to fast arithmetic, cryptography, combinatorics. Elementary graph theory. Introduction to rings and fields. Finite fields: coding theory, Hamming and BCH codes.
A. Bjorner

18.075 Advanced Calculus for Engineers (A except II, VI, VIII, XII, XIII, XVI, XVIII, XXII)

Prereq.: 18.03
G (1, 2, S)
3-0-9

Functions of a complex variable; calculus of residues. Ordinary differential equations; integration by power series; Bessel and Legendre functions. Expansion in series of orthogonal functions, including Fourier series. 18.075 and 18.04 may not both be taken for credit. Information. D. A. Vogan.

18.076 Advanced Calculus for Engineers (A except II, VI, XVI, XVIII, XXII)

Prereq.: 18.075
G (1, 2, S)
3-0-9

Vector analysis: orthogonal curvilinear coordinates. Calculus of variations. Solution of classical partial differential equations of mathematical physics, including applications of conformal mapping and the Laplace transformation. Information: D. A. Vogan.

18.085 Mathematical Methods for Engineers I (A) (Revised Content)

Prereq.: 18.03, 18.06
G (1, 2)
3-0-9

Review of linear algebra, applications to networks, structures, and estimation, Lagrange multipliers, differential equations of equilibrium, Laplace's equation and potential flow, boundary value problems, minimum principles and calculus of variations, Fourier series, transforms, convolution, complex variables, finite difference methods for partial differential equations, finite element methods.
G. Strang

18.086 Mathematical Methods for Engineers II (A) (New)

Prereq.: 18.03, 18.06
G (2)
3-0-9

Fast Fourier Transform and convolution, application to Poisson's equation, spectral methods, solution of linear and nonlinear systems and eigenvalue problems, initial-value problems for ordinary and partial differential equations, wave equation vs heat equation, von Neumann stability of difference equations, conservation laws, network flows, linear programming, simplex method and Karmarkar's method, duality and optimization.
G. Strang

18.089 Review of Mathematics

Prereq.: —
G (S)
Arr.

Reviews calculus and differential equations. Primarily for students in Course XIII-A. Degree credit allowed only in special circumstances.
D. A. Vogan

18.093 Tutoring in Mathematics

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (1, 2)
Arr.

For undergraduates who are teaching or tutoring in mathematics subjects. Limited enrollment, based on positions available. Permission must be secured in advance to register for this subject.
D. A. Vogan

18.099 Independent Activities

Prereq.: —
U (1, J, 2)
Arr.

For undergraduates desiring credit for studies during IAP or for special individual reading on an undergraduate level during the regular terms. Specific programs and credit arranged in consultation with individual faculty members and subject to departmental approval.
J. R. Munkres

Analysis

18.100 Analysis I (A except XVIII)

Prereq.: 18.03
U (1, 2) G (1, 2)
3-0-9

Two options offered, both covering fundamentals of mathematical analysis: convergence of sequences and series, continuity, differentiability, Riemann integral, sequences and series of functions, uniformity, interchange of limit operations. Both options show the utility of abstract concepts and teach understanding and construction of proofs. *Option A* chooses less abstract definitions and proofs, and gives applications where possible. *Option B* is more abstract and is for students with more mathematical maturity; places greater emphasis on point-set topology.

Option A: Term 1: *R. M. Dudley*
Term 2: *E. A. Carlen*
Option B: Term 1: *I. E. Segal, L. D. Saper*
Term 2: *F. P. Peterson, L. D. Saper*

18.101 Analysis II (A except XVIII)

Prereq.: 18.100, 18.701 or 18.710
U (1) G (1)
3-0-9

Continues 18.100, stressing the topics most useful in the study of manifolds and global analysis: differentiable maps, Jacobians, differentials, inverse and implicit function theorems, n-dimensional Riemann integral, change of variables in multiple integration, differential forms, general version of Stokes' theorem.
J. R. Munkres

18.102 Analysis II (A except XVIII)

Prereq.: 18.100
G (2) **Next offered 1986-87**
3-0-9

Continues 18.100, in the direction of modern integration theory and its applications. Lebesgue integration in Euclidean space. Its applications to Fourier analysis, including the Riesz-Fischer theorem. Brief introduction to functional analysis. Much more concrete treatment than in 18.125, 18.126, which deal with abstract measure spaces rather than Euclidean space.
I. E. Segal

18.103 Fourier Analysis — Theory and Applications (A except XVIII)

Prereq.: 18.100
G (1)
3-0-9

Continues 18.100. Roughly half the subject devoted to the theory of the Lebesgue integral and half to Fourier series and Fourier integrals. 18.103 is an alternative to 18.102; the material is somewhat similar, but it differs from 18.102 in that a heavy stress is paid to applications, particularly applications in probability theory.
V. W. Guillemin

18.104 Seminar in Analysis

Prereq.: 18.100
U (1)
3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Students report on and discuss topics taken from current journals or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite. 1985-86: Prime number theorem, density theorems, and primes in arithmetic progressions discussed.
N. C. Ankeny

18.115 Functions of a Complex Variable (A)

Prereq.: 18.100
G (1)
3-0-9

Exponential and trigonometric functions, Cauchy integral formula, holomorphic and meromorphic functions. Infinite series and products, the gamma function. Harmonic functions, conformal mapping, Dirichlet's problem.
W. M. Goldman

18.116 Topics in Complex Variables (A)

Prereq.: 18.115
G (2)
3-0-9

Topics vary from year to year: may be repeated for credit. Topics in the past: classical theory of automorphic functions; modular forms; Riemann surfaces; analytic number theory. Emphasizes as elementary an exposition as possible.
W. M. Goldman

18.117 Topics in Several Complex Variables (A)

Prereq.: 18.115, 18.125
G (1) **Next offered 1986-87**
3-0-9

Cauchy integral formula in polydiscs. Domains of holomorphy, pseudoconvexity and plurisubharmonic functions. Existence and approximation theorems for holomorphic functions via L^2-estimates for the $\bar{\partial}$-operator. Domains of holomorphy. $\bar{\partial}$-Neumann problem. Information: R. B. Melrose.

18.125 Measure and Integration (A)

Prereq.: 18.100
G (1)
3-0-9

Basic set theory and general topology (review and extension). Measures on rings of sets in abstract spaces; construction of invariant and probability measures. Abstract Lebesgue integration theory. The classical function spaces; introduction to Banach and Hilbert spaces. Convolution theory on groups.
I. E. Segal

18.126 Functional Analysis (A)

Prereq.: 18.125
G (2)
3-0-9

General theory of topological linear spaces, especially Banach and Frechet spaces; distributions, Banach algebras. Harmonic analysis in R^n, abelian, and compact groups. Spectral theory; self-adjoint operators; Stone's theorem and infinitesimal group representations. Applications to differential operators and equations. The Laplace transform and complex Fourier analysis; interpolation theory. Abstract probability and ergodic theory; functional integration, Wiener space. C*- and W*-algebras.
I. E. Segal

18.128 Geometric Measure Theory (A)

Prereq.: 18.125
G (2)
3-0-9

Hausdorff measure, rectifiable sets, structure theory, integral currents, compactness theorem. Applications to minimal surfaces, existence and regularity. Reports and discussions by students.
F. Morgan

18.135 Geometric Analysis (A)

Prereq.: 18.125
G (2)
3-0-9

Harmonic analysis on R^n. Spherical harmonics. Non-Euclidean Fourier analysis. Paley-Wiener type theorems, group-theoretic potential theory. Eigenfunctions, entire functionals and hyperfunctions. Radon transforms and applications.
S. Helgason

18.137 Harmonic Analysis (A) (New)

Prereq.: 18.155
G (2)
3-0-9

Applications of Fourier analysis to partial differential equations. Estimates in L^p, BMO, and other spaces for singular integral operators and Fourier multipliers. Applications to existence and uniqueness of solutions of nonlinear partial differential equations and linear partial differential equations with borderline smoothness. Exotic pseudodifferential operators.
D. S. Jerison

18.155 Distributions and Differential Equations (A)

Prereq.: 18.102
G (1)
3-0-9

Treats the basic theory of distributions with many applications to linear ordinary, and partial, differential equations: Distributions and elementary operations, Green's formula, wave operator in two dimensions. Fourier transform, temperate distributions, Sobolev spaces and constant coefficient elliptic operators. Convolution, fundamental solutions and the Malgrange-Ehrenpreis theorem. Schwartz kernel theorem.
A. Sanchez

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18.156 Introduction to Microlocal Analysis (A)

Prereq.: 18.155, 18.965
G (1) **Next offered 1986-87**
3-0-9

Examines singularities of distributions. Distributions singular across a submanifold, singular points of ordinary differential equations, non-characteristic boundary value problems. Pseudodifferential operators, regularity of elliptic differential operators, wavefront set and microdistributions. Darboux's theorem. Hamilton-Jacobi theory, the Maslov bundle. Lagrangian distributions, Fourier integral operators and the Cauchy problem for hyperbolic equations.
R. B. Melrose

18.157 Partial Differential Equations (A)

Prereq.: 18.155, 18.156
G (2)
3-0-9

Examines general classes of operators and problems in the theory of linear partial differential operators. Boundary value problems for elliptic operators, hypoelliptic operators with double characteristics. Operators of real principal type. Uniqueness for the Cauchy problem. Spectral theory.
R. B. Melrose

18.158 Topics in Differential Equations (A)

Prereq.: 18.125
G (2) **Next offered 1986-87**
3-0-9

Content varies from year to year; may be repeated for credit. Topics: Nonlinear hyperbolic equations, propagation of singularities, and interaction of nonlinear waves.
R. B. Melrose

18.159 Nonlinear Functional Analysis (A) (New)

Prereq.: 18.103 or 18.125
G (1)
3-0-9

Introduction to general nonlinear functional analysis (Sobolev spaces, convex analysis, Mountain pass lemma, etc.) with applications to geometry and physics: 1) Variational problems involving lack of compactness; Yamabe's and Rellich's conjectures; 'Large' harmonic maps. 2) Nonlinear elliptic and parabolic equations; isolated singularities; the Thomas-Fermi equation. 3) Periodic solutions for nonlinear wave equations.
H. Brezis

18.168 Analysis on Lie Groups and Homogeneous Spaces (A)

Prereq.: 18.755
G (2)
3-0-9

Invariant measures and abstract integral geometry illustrated by examples. Invariant differential operators and geometric transformations of these, like projections, radial parts and transversal parts. Global and local solvability, integral formulas for eigenfunctions and irreducibility questions for eigenspace representations.
S. Helgason

18.175 Theory of Probability (A)

Prereq.: 18.125
G (2) **Next offered 1986-87**
3-0-9

Ergodic theorems, laws of large numbers, convergence of probability measures, central limit theorems, stochastic processes. Brownian motion, martingales, strong Markov properties.
R. M. Dudley

18.177 Stochastic Processes (A)

Prereq.: 18.175
G (1, 2)
3-0-9

Topics in stochastic processes, such as Gaussian, Markov, diffusion and empirical processes. Content varies from year to year; may be repeated for credit.
Term 1: *D. W. Stroock*
Term 2: *R. M. Dudley*

18.199 Graduate Analysis Seminar (A)

Prereq.: Permission of Instructor
G (1)
3-0-21

Studies original papers in differential analysis and differential equations. Intended for first- and second-year graduate students. Permission must be secured in advance. Information: R. B. Melrose.

18.255 Mathematical Theory of Quantum Fields (A)

Prereq.: 18.101 or 8.022
G (2) **Next offered 1986-87**
3-0-9

Free quantum fields: their particle and functional integration representations. Unitary implementability of canonical transformations. C*-algebraic methods. Quantization of invariant wave equations in Minkowski and alternative space-times. Nonlinear local functions of fields: Wick products, introduction to constructive quantum field theory.
I. E. Segal

18.257 The Architecture of Fundamental Mathematical Physics (A)

Prereq.: Permission of Instructor
G (1) **Next offered 1986-87**
3-0-9

The basic principles of cosmology, elementary particle and quantum field theory, and general relativity, from a modern mathematical standpoint. For mathematicians who want an overview of fundamental theoretical physics, or for physicists who want to see how higher mathematics (C*-algebra, symmetries in space-time bundles, nonlinear partial differential equations, functional integration) is applicable. For integration and orientation; not a substitute for detailed technical courses.
I. E. Segal

18.284 Introduction to Functions of a Complex Variable (A except XVIII)

Prereq.: 18.03
G (1)
3-0-9

Complex numbers, analytic functions, Riemann surfaces for certain functions, Cauchy's theorem, singularities, residues, contour integrals, conformal mapping. Schwarz-Christoffel transformation, series and sequences, analytic continuation, harmonic functions, conjugate functions, the gamma function. More advanced than 18.04.
R. P. Stanley

18.295 General Relativity (A)

Prereq.: 18.06, 8.06, 8.312
G (2)
3-0-9

The physical background and mathematical formulation of the general theory of relativity. Includes tensor calculus, differential forms, and Riemannian geometry. Information: D. Z. Freedman.
N. P. Warner

Applied Mathematics

18.301 Introduction to Physical Mathematics I

Prereq.: 18.03
U (1)
3-0-9

Interdependence of mathematics and scientific problems, examples; deterministic and random processes; particle mechanics and differential equations, Brownian motion and random walk; Fourier analysis; tensors; partial differential equations of mathematical physics and continuum mechanics. Knowledge of 18.04 recommended.
D. Z. Freedman

18.302 Introduction to Physical Mathematics II

Prereq.: 18.301, 18.04 or 18.284
U (2)
3-0-9

Partial differential equations of mathematical physics, classification and solution of boundary-value problems, method of separation of variables. Sturm-Liouville problems, Fourier integrals, integral transforms. Bessel functions and other special functions. Applications of calculus of variations. A knowledge of complex variables desirable but not essential.
D. Kopriva

18.305 Methods of Applied Mathematics I (A)

Prereq.: 18.04 or 18.075 or 18.284 or 18.302
G (1)
3-0-9

18.306 Methods of Applied Mathematics II (A)

Prereq.: 18.04 or 18.075 or 18.284 or 18.302
G (2)
3-0-9

A comprehensive treatment of the aovanced methods of applied mathematics. Term 1: asymptotic behavior of differential and difference equations; asymptotic evaluation of integrals; regular and singular perturbation methods; boundary-layer techniques; WKB method; multiple scales. Term 2: partial differential equations; transform methods; characteristics, initial and boundary-value problems; Green's functions; singular perturbation problems; nonlinear wave propagation.
18.305: *A. Toomre*
18.306: Information: A. Toomre.

18.307 Methods of Applied Mathematics III (A)

Prereq.: 18.04 or 18.075 or 18.284 or 18.302
G (1) **Next offered 1986-87**
3-0-9

Selection of material from the following topics: calculus of variations (the first variation and the second variation). Integral equations (Volterra equations; Fredholm equations, the Hilbert-Schmidt theorem); the Hilbert Problem and singular integral equations of Cauchy type; Wiener-Hopf Method and partial differential equations; Wiener-Hopf Method and integral equations; group theory.
H. Cheng

18.308 Wave Motion (A)

Prereq.: 18.306
G (1)
3-0-9

Linear dispersive waves, group velocity. Near linear and nonlinear generalizations: modulation theory, average Lagrangian. Solitons, inverse scattering transform. Applications to water waves and nonlinear optics. Interaction of waves and streams, hydrodynamic stability. Self-gravitating fluid dynamics, density waves as dispersive waves, spiral galaxies as fluid systems and as stellar systems. Waves in compressible media, characteristics, shocks. Combustion theory: detonation waves, flames, stability theory, and bifurcation phenomena.
C-C. Lin, R. R. Rosales, G. Bertin

18.310 Principles of Applied Mathematics

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (1)
3-0-9

Introductory survey of fundamental concepts in applied mathematics: optimization, random process, coding, computer algorithms. This independent half of the complete sequence emphasizes the ideas and topics that relate to a "discrete" mathematical approach: computation, combinatorics, probability, linear programming.
M. Haiman, D. J. Kleitman

18.311 Principles of Applied Mathematics

Prereq.: 18.03
U (2)
3-0-9

Introductory survey of fundamental concepts in applied mathematics: propagation, stability, equilibrium, optimization. This independent half of the complete sequence emphasizes the ideas and topics that relate to a "continuous" mathematical approach, but connection with discrete mathematical approach also stressed: random walk, diffusion, waves, instabilities, characteristics and first order partial differential equations, with applications to traffic problems, fluid flow, and other problems in classical mathematical physics.
R. R. Rosales

18.313 Probability

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (2) SD
4-0-8

Development of theory and applications of probabilistic concepts for scientists and engineers. Emphasizes formulation and solution of probabilistic problems by the algebra of random variables. Topics: sample space, Bernoulli and Poisson processes, uniform process, generating functions and Laplace transforms, discrete and continuous-parameter Markov chains. Introduces the Central Limit Theorem and the foundations of probability.
G-C. Rota

18.314 Applied Combinatorial Analysis

Prereq.: 18.001 or 18.01 or 18.011 or 18.012
U (1)
3-0-9

Applications of combinatorial methods to practical problem solving. Emphasizes problems involving discrete optimization. Techniques from graph theory, matching theory, network flows. Other topics include enumeration, sorting and coding. Information: R. P. Stanley.

18.315 Combinatorial Theory (A)

Prereq.: Permission of Instructor
G (1)
3-0-9

Content varies from year to year; may be repeated for credit. Topics 1985-86: enumeration, generating functions, partially ordered sets, Möbius functions. Algebraic structures in combinatorics.
D. J. Kleitman, G-C. Rota

18.316 Seminar in Combinatorics (A)

Prereq.: Permission of Instructor
G (2)
3-0-9

Content varies from year to year; may be repeated for credit. Readings from current research papers in combinatorics. Topic to be chosen and presented by the class.
D. J. Kleitman

18.318 Topics in Combinatorics (A)

Prereq.: Permission of Instructor
G (2)
3-0-9

Content varies from year to year; may be repeated for credit. Topics 1985-86: Topological methods in combinatorics.
A. Björner

18.325 Topics in Applied Mathematics (A)

Prereq.: Permission of Instructor
G (2)
3-0-9

Combustion Theory and Hyperbolic Systems. Structure of flames and detonation waves. Shocks, shock reflections, and interactions. Inverse scattering. Modulation Theory. Connections of exact solutions of nonlinear evolution equations with path integrals. Information: R. R. Rosales.

18.330 Introduction to Numerical Analysis

Prereq.: 18.03
U (2)
3-0-9

Introduces basic techniques for efficient solution of numerical problems in science and engineering. Root finding, integration, function approximations, differential equations, direct and iterative methods in matrix theory, optimization with constraints, analysis of numerical stability.
D. Kopriva

18.335 Numerical Methods of Applied Mathematics I (A)

Prereq.: 18.06
G (1)
3-0-9

18.336 Numerical Methods of Applied Mathematics II (A)

Prereq.: 18.302
G (2)
3-0-9

Advanced introduction to theory and application of numerical methods. Term 1: numerical linear algebra, including solution of linear systems of equations, least-squares computations, eigenvalue computations, and iterative methods (classical, conjugate gradient, and Lanczos). Term 2: numerical solution of differential equations, especially of time-dependent partial differential equations by finite-difference and spectral methods, together with the associated theory of accuracy, stability, and convergence.
L. N. Trefethen

18.354 Fluid Mechanics

Prereq.: 18.04 or 18.075 or 18.302
U (1)
3-0-9

18.355 Fluid Mechanics (A)

Prereq.: 18.354
G (2)
3-0-9

A study of the basic concepts of fluid dynamics: conservation laws of mass, momentum, and energy; equation of state; vorticity and circulation theorems; boundary layer theory; instability and transition; waves; compressible flows and shocks; convection. Rotating fluids, multiphase fluids, and other selected topics of current research interest.
H. P. Greenspan

18.356 Rotating Fluids (A)

Prereq.: 18.305, 18.354
G (1) **Next offered 1986-87**
3-0-9

General theory of rotating fluids; transient flows; effects of viscosity, stratification, and compressibility; wave motion; nonlinear interactions; applications to centrifuge technology and the centrifugal separation of mixtures. Basic knowledge of fluid mechanics required.
H. P. Greenspan

18.357 Seminar in Fluid Dynamics (A)

Prereq.: 18.354
G (1)
3-0-9

Frontiers of nonlinear stability theory. Bifurcation in dynamical systems and the onset of chaos. Extension to fluid and magnetofluid systems with applications in geophysics and astrophysics. Students report on current papers and/or approved projects.
W. V. R. Malkus

18.358 Hydrodynamic Stability and Turbulence (A)

Prereq.: 18.354
G (2)
3-0-9

Linear stability theory of incompressible and compressible flows. Nonlinear stability theory; modes of transition, the advent of aperiodicity. Upper bound and statistical theories of turbulence. Statistical stability and the turbulent transport of heat and momentum. Properties of convection and shear turbulence.
W. V. R. Malkus

18.375 Stellar Dynamics and Galaxies (A)

Prereq.: 8.06, 18.076, or 18.302
G (2) **Next offered 1986-87**
3-0-9

Types and composition of galaxies, spiral arm and star formation. Dynamics of stellar systems and self-gravitating gaseous systems. Gravitational instability, density waves, dispersion relationship, resonances, self-excited spiral modes. Shock waves and star formation. Global modes, dynamical mechanisms for their excitation and maintenance. Simulation of real galaxies.
C-C. Lin

18.395 Group Theory with Applications to Physics (A)

Prereq.: 18.302 or 18.305 or 8.321
G (1) **Next offered 1986-87**
3-0-9

Selection of topics from the theory of finite groups, Lie groups, and group representations presented with some applications to quantum mechanics and particle physics.
D. Z. Freedman

18.396 Topics in Theoretical Physics (A)

Prereq.: 8.323
G (1, 2) **Next offered 1986-87**
3-0-9

Topics vary from year to year; may be repeated for credit. Term 1: Quantum Field Theory. Term 2: Survey of supersymmetric quantum field theories including both rigid (global supersymmetry) and local (supergravity) models. Both component approaches and the superspace-superfield formalism are treated. Information: D. Z. Freedman.

18.400J Automata, Computability, and Complexity

(18.420J)
(Same subject as 6.045J)
Prereq.: 18.063 or 18.310
U (1, 2)
3-0-9

See description under subject 6.045J.
Term 1: *A. R. Meyer*
Term 2: Information: M. Sipser.

18.404J Theory of Computation (A except XVIII) (Revised Content)

(18.427J)
(Same subject as 6.840J)
Prereq.: 18.063 or 18.310
U (1)
3-0-9

A more extensive and theoretical treatment of the material in 6.045J/18.400J emphasizing computability, computational complexity, and mathematical logic. Reducibility, degrees of unsolvability, Kolmogorov complexity. Time and space measures on computation, efficient reducibility, complete problems. Decidable, undecidable, and inherently complex logical theories.
M. Sipser

18.405J Advanced Complexity Theory (A)

(18.428)
(Same subject as 6.841J)
Prereq.: 6.840J/18.404J
G (2)
3-0-9

Current research topics in computational complexity theory. Nondeterministic, alternating, probabilistic, and parallel computation models. Boolean circuits. Complexity classes and complete sets. The polynomial-time hierarchy. Relativization. Definitions of randomness. Approaches to the P = ?NP and related questions.
M. Sipser, A. R. Meyer

18.410J Introduction to Algorithms (New)

(Same subject as 6.046J)
Prereq.: 6.001, 18.063 or 18.310
U (1)
3-0-9

See description under subject 6.046J.
R. L. Rivest

18.414J Theory of Algorithms (A except XVIII) (Revised Content)

(18.437J)
(Same subject as 6.851J)
Prereq.: 18.06 or 18.710, 18.063 or 18.310
U (2)
3-0-9

Techniques for design and analysis of algorithms, emphasizing mathematical methods and proofs. Proof-oriented version of 6.046J/18.410J. Topics: Data structures, sorting, selection, hashing. Solving recurrences. Upper and lower bounds. Dynamic programming. Divide and conquer. Graph algorithms: spanning

trees, matching, shortest paths, max flow. Matrix operations. Fast Fourier transform. Integer and polynomial arithmetic. Permutation group membership. Primality testing. Linear programming. Parallel algorithms.
F. T. Leighton

18.415J Advanced Algorithms (A) (Revised Content)

(18.438)
(Same subject as 6.854J)
Prereq.: 6.851J/18.414J
G (1)
3-0-9

Continuation of 6.851J/18.414J, emphasizing fundamental algorithms and advanced methods of algorithmic design and analysis. Topics: Linear programming. Matroid algorithms, maximum matching. Basis reduction, integer programming, polynomial factorization, diophantine approximation. Cryptography and randomness. Distributed algorithms, Byzantine Generals. Parallel algorithms and VLSI computation, universal simulations, area-time trade-offs. Computational geometry. Probabilistic analysis of algorithms. Advanced data structures. Information: F. T. Leighton.

18.419 Topics in Algorithms and Complexity (A)

(18.436)
Prereq.: Permission of Instructor
G (2)
3-0-9

A seminar on advanced topics in algorithms and complexity. Current literature presented by students and instructors with a view toward preparing students for research in theory of algorithms and complexity. May be repeated for credit. Information: F. T. Leighton.

18.421J Algorithmic Algebra and Number Theory

(18.411)
(Same subject as 6.047J)
Prereq.: 18.06 or 18.710, 18.063 or 18.310 or 18.703
U (2) **Not to be offered 1986-87**
3-0-9

Emphasis on constructing efficient algorithms for classical problems in algebra and number theory. Integer and polynomial GCD computation, modular arithmetic, chinese remainder theorem, Jacobi symbol computation, primality testing, extracting square roots mod primes, integral lattices, factorization of polynomials over the rationals, simultaneous diophantine approximations, solving binary quadratic and cubic modular equations, application to public-key cryptography. Alternate years. Information: F. T. Leighton.

18.425J Cryptography and Cryptanalysis (A) (New)

(Same subject as 6.875J)
Prereq.: 6.046J/18.410J or 6.851J/18.414J or 6.047J/18.421J
G (2)
3-0-9

See description under subject 6.875J.
S. Micali

18.426J Advanced Topics in Cryptography (A) (New)

(Same subject as 6.876J)
Prereq.: Permission of Instructor
G (1)
3-0-9

See description under subject 6.876J.
S. Goldwasser

18.427J Program Semantics and Verification (A) (New)

(Same subject as 6.830J)
Prereq.: 6.821, 6.045J/18.400J or 6.840J/18.404J
G (2) **Next offered 1986-87**
3-0-9

See description under subject 6.830J.
A. R. Meyer

18.429J Algebraic Manipulation (A) (New)

(Same subject as 6.856J)
Prereq.: 6.821, 18.063 or 18.310
G (2) **Next offered 1986-87**
3-0-9

See description under subject 6.856J.
J. Moses, R. E. Zippel

18.431J Graph Algorithms (Revised Content)

(18.424)
(Same subject as 6.048J)
Prereq.: 18.063 or 18.310 or 18.314
U (1) **Next offered 1986-87**
3-0-9

Emphasizes the design of efficient algorithms for graph theoretic problems. Includes fast algorithms for: Shortest-paths problems, minimum spanning trees, depth-first search and graph decompositions, network flow algorithms, networks with upper and lower bounds, vertex and edge connectivity, maximum cardinality matchings, maximum weight matchings, planarity testing, distributed graph algorithms. Alternate years. Information: F. T. Leighton.

18.435 Theory of Parallel Computation and VLSI (A)

(18.439)
Prereq.: 6.046J/18.410J or 6.851J/18.414J
G (1)
3-0-9

Introduces parallel computation and very large scale integration. Design and analysis of systolic algorithms for sorting, Fourier transform and arithmetic on arrays, hypercubes and other fixed-connection networks. Network transformations, broadcast simulation, retiming. Lower bounds, completeness results. Mathematical models of chips. Area-time and other tradeoffs. Placement and routing. Schemes for minimizing propagation delays, wire crossings, and layout area. Possible topics: fault-tolerance, testing, 3D-VLSI, wafer-scale systems and supercomputers.
F. T. Leighton

18.437J Distributed Algorithms (A) (New)

(Same subject as 6.852J)
Prereq.: Permission of Instructor
G (1) **Next offered 1986-87**
3-0-9

See description under subject 6.852J. Information: N. A. Lynch.

Applied Mathematics: Statistics

18.440 Probability and Random Variables

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (1, 2) SD
4-0-8

Topics in applications. Probability spaces, random variables, distribution functions, expected value. Binomial, geometric, hypergeometric, Poisson distributions. Uniform, exponential, normal, gamma and beta distributions. Mean, variance, moments and generating functions. Conditional probability, Bayes theorem, joint distributions, and distributions of transformed random variables. Tchebychev inequality, law of large numbers and central limit theorem. Multivariate normal distribution, covariances and correlation. Applications to statistics and decision theory.
Term 1: *D. Riceman*
Term 2: *R. M. Dudley*

18.441 Statistical Inference (A except XVIII)

Prereq.: 6.041 or 18.440 or 18.313
U (2) G (2)
4-0-8

Introduces statistical inference. Decision theory, hypothesis testing, point and interval estimation. Bayesian methods, maximum-likelihood and likelihood-ratio tests. Chi-square goodness of fit tests. Comparison of populations by parametric and nonparametric methods. Analysis of variance, regression, and correlation. Sequential analysis if time permits. Treatment more mathematical than that of 18.05 and more detailed in its treatment of statistics.
L. Kavalieris

18.443 Statistics for Applications (A except XVIII)

Prereq.: 18.440 or 18.313 or 6.041
G (1)
4-0-8

A broad treatment of statistics concentrating on specific statistical techniques used in science and industry. Topics: hypothesis testing and estimation. Chi-square goodness of fit, regression, correlation, analysis of variance and experimental design. Treatment more oriented toward application and less toward theory than 18.441. Information: R. E. Welsch.

18.444 Probability and Statistics for Scientists and Engineers (A)

Prereq.: 18.002 or 18.02 or 18.021 or 18.022, 18.06
G (1)
Arr.

Accelerated introduction covering material in 18.440, 18.441 and part of 18.445J; for engineers and scientists talented mathematically and who need to apply statistical ideas but lack time for two or three semesters of course work. Case study point of view. Topics: probability, distribution expectation, central limit theorem, law of large numbers, estimation, hypothesis testing, confidence intervals, maximum likelihood, information, Neyman-Pearson theory, regression, design of experiments, decision theory, and Markov processes.
H. Chernoff

18.445J Introduction to Stochastic Processes (A)

(Same subject as 15.073J)
Prereq.: 18.313 or 18.440 or 6.041
G (1, 2)
4-0-8

Introduces the theory and application of stochastic processes. Empirical phenomena for which stochastic processes provide models. Random walk, gambler's ruin, recurrent events, discrete-time Markov chains, Branching processes, Poisson processes, renewal theory. Continuous-time Markov chains, birth and death processes, queueing theory, and Brownian motion.
Term 1: *D. Riceman*
Term 2: *A. I. Barnett*

18.446 Applied Time Series Analysis (A)

Prereq.: 18.441 or 18.443 or 15.075
G (1)
4-0-8

Statistical techniques commonly used to analyze time series data. Topics: estimation of trends and seasonal adjustment, stationary series — autocorrelation and spectrum. Estimation and interpretation of spectra. ARIMA models and fitting them to data. Analysis of bivariate series — cross correlation and cross spectrum. Emphasizes learning techniques by using them on actual data.
L. Kavalieris

18.448 The Analysis of Categorical Data (A)

Prereq.: 18.441 or 18.443 or 15.075
G (1)
4-0-8

Theory and application of log-linear models to multi-way contingency tables and other data sets where the dependent variable is categorical. Topics: the Poisson distribution, one-way, two-way, and multi-way frequency tables, logit regression, and maximum likelihood estimation and computations. Class uses and compares various computer programs for the estimation of log-linear models. Most of the technical theoretical material is reserved until the final few weeks of the semester, and students outside of Course XVIII may arrange to substitute a data analysis project for it. Information: H. Chernoff.

18.453 Quality Control and Reliability (A)

Prereq.: 18.440; 18.441 or 18.443 or 15.075
G (1)
3-2-7

Introduces statistical concept in quality control and reliability analysis. Topics: statistical design of Shewhart and CUSUM control charts; acceptance sampling, probabilistic representations and analyses of lifetime data; measurement and prediction of reliability characteristics. Interactive statistical graphics system used to demonstrate the methodology developed in the subject, as well as to provide a student laboratory environment.
M. Schatzoff

18.454 Sampling, Simulation, and Monte Carlo (A)

Prereq.: 18.440 or 18.313 or 6.041
G (2)
4-0-8

Introduction to principles and techniques of sampling for the purpose of a survey. Including: simple random sampling, stratified sampling, systematic sampling, and cluster sampling. Discussion of statistical background of Monte Carlo methods and simulation — prominent parts of experimental mathematics with wide applicability. Including: variance reduction, conditional Monte Carlo, control variates, antithetic variates, regression methods, Monte Carlo optimization, application to statistical inference problems.
H. Chernoff

18.455 Analysis of Variance and Design of Experiments (A)

Prereq.: 18.06, 18.441 or 18.443 or 15.075
G (1)
4-0-8

Detailed presentation and use of the classical models of analyses of variance (ANOVA): one-way classification, two-way classifications, block designs, nested designs, latin squares, etc. Model II type of designs. Tests of hypothesis, simultaneous confidence intervals. Presentation of regression, and analysis of covariance (ANOCOVA). Properties of the multivariate normal and related distributions, linear models, general linear hypothesis. Geometric interpretation. Finally: effect of departure from assumptions and nonparametric analogs of ANOVA.
L. Kavalieris

18.456 Multivariate Methods in Statistics (A)

Prereq.: 18.06; 18.441 or 18.443 or 15.075
G (2)
4-0-8

Theory and application of commonly used techniques involving multivariate data. Attention devoted to specific applications, and to computational facilities for applying the methods. Selects topics from the following: multivariate regression, discriminant analysis, and pattern classification. Cluster analysis, factor analysis, and principal components. Multidimensional scale analysis. Contingency tables.
M. A. Wong

18.457J Statistics for Model Building (A)

(Same subject as 15.076J)
Prereq.: 18.06; 18.443 or 15.075
G (2)
3-0-9

See description under subject 15.076J.
R. E. Welsch

18.458 Robust Statistics and Nonparametric Methods (A)

Prereq.: 18.440; 18.441 or 18.443 or 15.075
G (1) **Next offered 1986-87**
3-0-9

Overview of robust statistical theory, including: asymptotic minimax, infinitesimal (bounded-influence) aspects, robust covariances and robust regression. Nonparametric methods which give useful and valid results under a very wide class of underlying distributions — particularly useful for social scientists and biologists. Topics: Wilcoxon test, sign test, Wilcoxon-Mann-Whitney test, U-statistics theorems, optimal linear rank tests, Kruskal-Wallis test, rerandomization tests. Information: R. E. Welsch.

18.459 Statistical Laboratory (A)

Prereq.: Permission of Instructor
G (2) **Next offered 1986-87**
2-0-4

Lectures consist of presentations of applied problems with or without solutions. Students give a presentation based on the literature or the solution of a minor problem. The solution of a major problem serves as the basis for a master's thesis. Information: H. Chernoff.

18.465 Topics in Statistics (A)

Prereq.: Permission of Instructor
G (1)
3-0-9

Introduces some special theoretical topics in mathematical statistics at an intermediate level. Assumes familiarity with the elements of probability theory and the fundamental concepts and basic techniques of statistical inference. Topics chosen in accordance with the interests of the instructor and the students. Information: H. Chernoff.

18.466 Mathematical Statistics (A)

Prereq.: 18.441 or 18.443
G (1)
3-0-9

Decision theory, estimation, confidence intervals, hypothesis testing. Introduces large sample theory. Asymptotic efficiency of alternative statistical procedures. Linear statistical inference. Permission of instructor required.
H. Chernoff

18.467 Mathematical Statistics (A)

Prereq.: 18.466, 18.102 or 18.125
G (2) **Next offered 1986-87**
3-0-9

Continues 18.466, using elementary measure theory. Sequential analyses. Permission of instructor required. Information: L. Kavalieris.

18.468 Advanced Time Series Analysis (A)

Prereq.: 18.441 or 18.443
G (2) **Next offered 1986-87**
4-0-8

Stationary vector processes. Correlation. Spectra. Vector autoregressive-moving average models — parameterisation and estimation. The Kalman filter. Depending on participants' interests, topics chosen from: robust filtering and parameter estimation; incomplete data problems; model order determination; nonlinear models. Permission of instructor required. Information: L. Kavalieris.

18.475 Current Topics in Statistics (A)

Prereq.: Permission of Instructor
G (2)
3-0-6

Introduces current topics in applied mathematical statistics at an intermediate level. Assumes students are familiar with the elements of probability theory and the fundamental concepts and basic techniques of statistical inference. Typically covers several topics connected by a common theme.
H. Chernoff

Logic

18.504 Seminar in Logic

Prereq.: —
U (1) **Next offered 1986-87**
3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Students report on and discuss topics taken from current journals or from texts not regularly used in other mathematics subjects.
S. D. Friedman

18.511 Introduction to Mathematical Logic

Prereq.: —
U (2)
3-0-9

Propositional and predicate logic. Elementary model theory, completeness, compactness, and Lowenheim-Skolem theorems; elementary recursion theory, enumeration and recursion theorems. Gödel incompleteness theorems. Special additional topics as time permits. While this subject has no formal prerequisite, any Course XVIII subject with first decimal digit one or higher is adequate preparation. Information: C. G. Bailey.

18.515 Mathematical Logic (A)

Prereq.: 18.511
G (1)
3-0-9

18.516 Mathematical Logic (A)

Prereq.: 18.515
G (2)
3-0-9

First-order logic. Completeness, compactness, Lowenheim-Skolem theorems. Type-omitting, categoricity. Incompleteness of arithmetic. Recursively enumerable sets and Turing degrees. Friedberg-Muchnik theorem. Zermelo-Fraenkel set theory. Ordinals and cardinals. Reflection principles, absoluteness. Consistency of axiom of choice and Generalized Continuum Hypothesis. Permission of instructor required for those not having 18.511.
18.515: *M. S. Legrand*
18.516: *C. G. Bailey*

18.565 Recursion Theory (A)

Prereq.: 18.516
G (2)
3-0-9

Topics in recursion theory chosen from: priority arguments, hyperarithmetic theory, ordinal recursion, E-recursion, theory of projective sets. Permission of instructor required for those not having 18.516.
G. E. Sacks

18.575 Theory of Models (A)

Prereq.: 18.516
G (1) **Next offered 1986-87**
3-0-9

Topics in model theory chosen from: model-theoretic algebra, Morley rank, infinitary model theory, generalized quantifiers, ultraproducts. Permission of instructor required for those not having 18.516.
S. D. Friedman

18.585 Set Theory (A)

Prereq.: 18.516
G (2) **Next offered 1986-87**
3-0-9

Forcing. Other topics in set theory chosen from: large cardinals, combinatorial set theory, iterated forcing, descriptive set theory, fine structure of L. Permission of instructor required for those not having 18.516.
S. D. Friedman

18.595 Seminar on Current Topics in Logic (A)

Prereq.: 18.565, 18.575, 18.585
G (1, 2)
3-0-9

Analysis of results of current interest in logic. Students present recent developments in the field for general discussion. Uses formal and informal sources. Topics vary from year to year; may be repeated for credit.
Term 1: *G. E. Sacks*
Term 2: *C. G. Bailey*

18.597 Universal Algebra (A) (New)

Prereq.: Permission of Instructor
G (2)
3-0-9

Topics vary from year to year; may be repeated for credit. Properties shared by algebraic structures, introduction to category theory and lattice theory, elementary equivalence of algebraic structures, properties preserved under algebraic constructions, equational logic, word problems.
M. Haiman, G-C. Rota

Algebra and Number Theory

18.701 Algebra I

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (1)
3-0-9

18.702 Algebra II

Prereq.: 18.701
U (2)
3-0-9

More extensive and theoretical than the 18.710-18.703 sequence. Experience with linear equations and matrices. First term: group theory, geometry, and linear algebra. Second term: rings and fields: ideals, polynomial rings, factorization, modules, Jordan form for matrices, extension fields, Galois theory.
M. Artin

18.703 Modern Algebra

Prereq.: 18.002 or 18.02 or 18.021 or 18.022
U (2) SD
3-0-9

A one-term treatment, covering the traditional algebra topics which have found greatest application in science and engineering as well as in other mathematical disciplines: group theory, emphasizing finite groups; ring theory, including ideals, unique factorization in polynomial and Euclidean rings; field theory, including properties and applications of finite fields. 18.710 and 18.703 together cover most of basic algebra. 18.06 or 18.710 should precede 18.703 if both subjects are to be taken.
R. D. Schafer

18.704 Seminar in Algebra and Number Theory

Prereq.: 18.702 or 18.703
U (2)
3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Students report on and discuss topics taken from current journals or from texts not regularly used in other mathematics subjects.
S. Kleiman

18.705 Commutative Algebra (A)

Prereq.: 18.701-18.702 or 18.710-18.703
G (1)
3-0-9

Basic topics in commutative algebra, with a brief introduction to categorical ideas and homological algebra. Modules, localization, noetherian rings, finiteness properties, dimension theory. Text: Atiyah and MacDonald.
S. Kleiman

18.706 Noncommutative Algebra (A)

Prereq.: 18.705
G (2) **Next offered 1986-87**
3-0-9

Topics in noncommutative algebra, selected from such areas as Wedderburn theorems, group representations, Lie algebras, algebraic groups. Information: S. Kleiman.

18.710 Abstract Linear Algebra

Prereq.: 18.002, 18.02 or 18.021 or 18.022
U (1) SD
3-0-9

An algebraic treatment of linear algebra including vector spaces, systems of linear equations, bases, linear independence, matrices, determinants, eigenvalues, inner products, quadratic forms, and canonical forms of matrices. Compared with 18.06, more emphasis on theory and proofs, less on matrix calculations and applications.
R. D. Schafer

18.711 Game Theory

Prereq.: —
U (1)
3-0-9

Two-person combinatorial games. Finding winning moves in such games as Nim, Hackenbush, and Dots-and-Boxes. Analysis of positions. Algorithmic and algebraic strategies for reduction of positions. Study of impartial games. Matrix games, continuous games, Arrow's Theorem. No formal prerequisite, but students should be familiar with elements of linear algebra. Information: N. C. Ankeny.

18.715 Topics in Homological Algebra (A)

Prereq.: 18.705
G (1) **Next offered 1986-87**
3-0-9

18.716 Topics in Homological Algebra (A)

Prereq.: 18.715
G (2) **Next offered 1986-87**
3-0-9

Content varies from year to year; may be repeated for credit. Topics selected from such areas as, cohomology of groups and Lie algebras, K-theory, characteristic classes, higher algebraic K-theory, cyclic cohomology.
D. G. Quillen

18.725 Algebraic Geometry (A)

Prereq.: 18.705
G (1)
3-0-9

Introduces contemporary algebraic geometry. Centered around the basic techniques of the subject, including sheaf cohomology and the Riemann-Roch theorem for curves.
S. Kleiman

18.727 Topics in Algebraic Geometry (A)

Prereq.: 18.725
G (2)
3-0-9

Topics vary from year to year; may be repeated for credit. Introduction to Hodge theory, primarily of smooth complex projective varieties. Includes: harmonic theory on a compact manifold, Hodge theorem, Hodge decomposition on a compact Kähler manifold, Lefschetz decomposition, Kodaira vanishing theorem and embedding theorem.
L. D. Saper

18.735 Topics in Algebra (A)

Prereq.: 18.966 or 18.725
G (1)
3-0-9

Content varies from year to year; may be repeated for credit. Topic for 1985-86: Algebraic Analysis. Introduction to systems of differential equations and systems of microdifferential equations on complex manifolds. Includes: canonical transformations, microfunctions, structure of holonomic systems, regular singularities, Riemann-Hilbert correspondence, and index theorems. Applications include prehomogeneous vector spaces.
K. K. Vilonen

18.737 Linear Algebraic Groups (A)

Prereq.: 18.705
G (1)
3-0-9

Introduces the classification of affine groups over an algebraically closed field via their representations as groups of invertible matrices.
M. Artin

18.739 Theory of Invariants (A)

Prereq.: 18.705
G (2) **Next offered 1986-87**
3-0-9

Hilbert's finiteness theorem for reductive groups. Properties of the orbits and the orbit space. Classical invariants theory. Hilbert-Mumford-Richardson theorem. Rosenlicht's theorem on the existence of invariants. Matsushima criterion. Richardson's theorem on the principal stabilizer. Chevalley-Luna-Richardson theorem. Linear actions with a non-trivial stabilizer. Nice representations. Methods of the orbit classification. Applications to classical problems of linear algebra. Other topics.
V. Kač

18.745 Introduction to Lie Algebras (A)

Prereq.: 18.755
G (2)
3-0-9

Emphasizes theory of Lie algebras, and algebraic aspects of Lie theory. Structure of finite-dimensional Lie algebras; Engel and Lie theorems, Cartan subalgebras, Cartan criteria. Structure and classification of semi-simple Lie algebras. Weyl and Levi theorems. Campbell-Hausdorff formula and connection between Lie groups and Lie algebras. Finite-dimensional representations of semi-simple Lie algebras, Weyl character formula. Verma modules.
D. A. Vogan

18.747 Infinite-dimensional Lie Algebras (A)

Prereq.: 18.745
G (2)
3-0-9

Kač-Moody Lie algebras. Highest weight representations. Character formulas. Applications to combinatorics and invariant theory. Connection with the theory of theta functions and modular forms. Vertex representations and their relation to soliton equations and to the quantum field theory. Virasoro algebra and the determinantal formula. Connection to critical exponents in statistical mechanics. Other topics.
V. Kač

18.748 Introduction to Infinite-dimensional Groups (A)

Prereq.: Permission of Instructor
G (1) **Next offered 1986-87**
3-0-9

Construction of groups associated to infinite-dimensional Lie algebras. Tits systems for groups associated to Kač-Moody algebras. Geometry of infinite-dimensional flag varieties. Infinite-dimensional analogs of compact Lie groups. Special functions of infinite-dimensional groups. Applications to the Korteweg-deVries equation and its relatives.
V. Kač

18.755 Introduction to Lie Groups (A)

Prereq.: 18.100, 18.710
G (1)
3-0-9

A broad general introduction to Lie groups, suitable for physicists as well as mathematicians. Study of basic elementary examples: the classical matrix groups, the Galilei, Lorentz, and Poincare groups. Spinors, special functions, invariants, classical dynamics. Introduction to manifolds, general Lie groups, homogeneous spaces, and Lie algebras. Automorphism and adjoint groups.
R. Zierau

18.756 Analysis on Lie Groups (A)

Prereq.: 18.755
G (2) **Next offered 1986-87**
3-0-9

Semi-simple Lie groups and symmetric spaces. Topics in function theory on symmetric spaces, such as Fourier analysis and Radon transform, invariant differential operators and potential theory. Emphasizes connections with classical analysis and representation theory.
M. F. Vergne

18.757 Representations of Lie Groups (A)

Prereq.: 18.755
G (2)
3-0-9

Lie groups, Lie algebras, and their representations. Induced representations. Representations of compact groups, nilpotent groups, and semi-simple groups. Analysis on semisimple Lie groups and symmetric spaces. Invariant differential operators and their role in representation theory.
B. Kostant

18.758 Representations of Lie Groups (A)

Prereq.: 18.757
G (2) **Next offered 1986-87**
3-0-9

Representations of semisimple Lie groups: theory of Harish-Chandra modules, Langlands' classification, Kazhdan-Lusztig conjectures. Decomposition of principal series representations, unitarity problem.
D. A. Vogan

18.769 Topics in Lie Theory (A)

Prereq.: Permission of Instructor
G (1)
3-0-9

Topics in Lie theory, varying from year to year. May be repeated for credit. Representations of Hecke algebras. Requires familiarity with Lie groups and Lie algebras.
B. Kostant

18.775 Algebraic Number Theory (A)

Prereq.: 18.705
G (1) **Next offered 1986-87**
3-0-9

18.776 Algebraic Number Theory (A)

Prereq.: 18.775
G (2)
3-0-9

Reviews basic algebraic number theory. Classical formulation of class field theory, including existence and uniqueness of class fields, the conductor-discriminant formula, Artin's reciprocity law. Artin L-series. Adelic formulation. The "simple algebra" approach. Applications to the study of arithmetic of cubic fields and (if time permits) to the theory of complex multiplication.
N. C. Ankeny

18.785 Analytic Number Theory (A)

Prereq.: 18.115
G (1)
3-0-9

Analytic algebraic number theory. Functional equations of zeta and L-functions. Zero-free regions and prime number theorems. Applications to density theorems in algebraic number theory including Artin's conjecture on primitive roots.
H. M. Stark

18.786 Topics in Number Theory (A)

Prereq.: Permission of Instructor
G (1) **Next offered 1986-87**
3-0-9

Topics vary from year to year; may be repeated for credit. Topics in the past few years include: Diophantine analysis and transcendence; quadratic number fields and complex multiplication; automophoric forms; diophantine geometry.
H. M. Stark

MIT 1985

Topology and Geometry

18.901 Introduction to Topology I (A except XVIII)

Prereq.: 18.100
G (1, 2)
3-0-9

18.902 Introduction to Topology II (A except XVIII)

Prereq.: 18.901, 18.701 or 18.703
G (2) **Next offered 1986-87**
3-0-9

Introduces topology, covering topics fundamental to modern analysis and geometry. Intended for those going on to graduate work. 18.901: topological spaces, connectedness, compactness, continuous functions, separation axioms, function spaces. Metrization theorems, the Tychonoff theorem. Topological groups. 18.902: introduction to algebraic topology. Fundamental group, covering spaces, Van Kampen theorem, classification of covering spaces. Applications to knot theory, classification of compact surfaces, separation theorems in the plane.
18.901: Term 1: *F. P. Peterson*
Term 2: *L. D. Saper*

18.904 Seminar in Topology

Prereq.: 18.901
U (2)
3-0-9

Seminars for mathematics majors in several topics, each under the direction of a faculty member whose special interest is in the field of the seminar. Students report on and discuss topics taken from current journals or from texts not regularly used in other mathematics subjects. Certain topics may require an additional prerequisite. Normally covers the same material as 18.902, but in seminar form.
D. M. Kan

18.905 Algebraic Topology (A)

Prereq.: 18.702 or 18.705; 18.901
G (1)
3-0-9

18.906 Algebraic Topology (A)

Prereq.: 18.905
G (2)
3-0-9

Fundamental group, covering spaces, simplicial homology, simplicial approximation manifolds. Homology and cohomology of topological spaces, universal coefficient theorem, plus additional topics to be chosen by the instructor.
D. J. Anick

18.915 Graduate Topology Seminar (A)

Prereq.: 18.906
G (1)
3-0-21

Study and discussion of important original papers in the various parts of algebraic and differential topology. Open to all students who have had 18.906 or the equivalent, not only prospective topologists.
D. M. Kan

18.917 Advanced Topology (A)

Prereq.: 18.906
G (2)
3-0-9

Content varying from term to term so that graduate students taking the subject in successive terms may have an introduction to several important phases of topology such as homotopy theory, cohomology theory, fibre spaces, K-theory, combinatorial topology, and/or differential topology. Homotopy theory, cobordism theory.
J. R. Munkres

18.950 Elementary Differential Geometry

Prereq.: 18.100, 18.901
U (2)
3-0-9

Simplicial complexes; manifolds; differential forms and the DeRham theorem; Riemannian geometry of surfaces; connections on principal bundles; curvature; the Gauss-Bonnet theorem. Permission of instructor required for those not having 18.100 and 18.901.
I. M. Singer

18.961 Elementary Differential Topology

Prereq.: 18.101, 18.901
U (2) **Next offered 1986-87**
3-0-9

Differentiable manifolds in R^n; differentiable mappings; transversality; Sard's theorem; intersection theory; the Euler characteristic of a manifold and the Lefschetz fixed point theorem; integration on manifolds, Stokes' theorem, DeRham cohomology.
V. W. Guillemin

18.965 Geometry of Manifolds (A)

Prereq.: 18.101
G (1)
3-0-9

18.966 Geometry of Manifolds (A)

Prereq.: 18.965
G (2)
3-0-9

Differentiable manifolds, vector fields and forms, introduction to Lie groups, the DeRham theorem, Riemannian manifolds. 18.966 continues 18.965. Focuses on symplectic geometry and classical dynamical systems.
V. W. Guillemin

18.969 Topics in Geometry (A)

Prereq.: 18.965
G (1)
3-0-9

Content varies from year to year; may be repeated for credit. Topics have included geometry of hyperbolic space; discontinuous groups of isometries of hyperbolic space; limit sets of Kleinian groups; uniformization; convexity; structure of discrete subgroups; rigidity; deformation theory (Teichmüller space, geometric convergence, hyperbolic Dehn surgery); ergodic properties of geodesic flow; existence of hyperbolic structures on 3-manifolds. Permission of instructor required for students not having 18.965.
W. M. Goldman

18.975 Elliptic Operators (A)

Prereq.: 18.965
G (1) **Next offered 1986-87**
3-0-9

18.976 Elliptic Operators (A)

Prereq.: 18.975
G (2) **Next offered 1986-87**
3-0-9

Elliptic pseudodifferential operators and Fourier integral operators associated with them. Spectral theorems relating asymptotic properties of the spectrum to geometric properties of the bicharacteristics. The Selberg trace formula and various generalizations of it. 18.976 continues 18.975. Spectral properties of differential and pseudodifferential operators on open manifolds, scattering theory, non-self-adjoint operators.
V. W. Guillemin

18.994 Seminar in Geometry

Prereq.: —
U (2) **Next offered 1986-87**
3-0-9

Seminars in several topics, for math majors. Each under direction of a faculty member whose interest is in the field of the seminar. Students report on and discuss topics from current journals or texts not regularly used in other math subjects. May be repeated for credit.
F. Morgan

18.999 Mathematical Reading

Prereq.: —
G (1, 2, S)
Arr.

Reading of advanced mathematical treatises under supervision of a member of the Department. For graduate students desiring advanced work not provided in regular subjects.
N. C. Ankeny