Cajori Two Course Inventory

Category 1. Elementary Courses in Algebra, Geometry and Trigonometry (Worksheet Abbreviation: ElemAlgGeoTrig)

Cluster[1]: Pre-college[2] mathematics

1.1 Arithmetic/Quantitative Reasoning.
Fractions, decimals, percents, graphs, areas and perimeters and volumes.

1.2 Elementary Algebra.
The first secondary school level course in algebra. Unknowns, linear equations, word problems, interest, etc.

1.3 Plane Geometry.
Elementary Euclidean geometry in the plane. Congruent and similar triangles, properties of circles, right triangles, the Pythagorean theorem.

1.4 Intermediate Algebra.
Polynomials, quadratic equations, 2x2 linear systems, ratio, etc.

1.5 Trigonometry.
The trigonometric functions and trigonometric identities.

1.6 Comprehensive Remedial Mathematics.
Review of elementary algebra and plane geometry and possibly arithmetic as well. This combines the material of earlier listed courses and will presumably have a brisker pace as it is a review of these individual subjects.

Cluster: Elementary college level geometry

1.7 Solid Geometry.
Euclidean solid geometry, via axioms. Points, lines, planes, spheres, some polyhedra.

1.8 Spherical Geometry.

[1] This grouping into clusters was a bit of an afterthought, hit upon when the software design and implementation were already well advanced. It is not possible to call up tabulations by cluster.
[2] As currently interpreted in the early 21st century.

The geometry of points and great circles on the sphere. Angles, areas, spherical excess. Applications to mapmaking and navigation. Note that a course by this title could also be used to explore the axiom set for spherical geometry, comparing it to the axioms of Euclidean Geometry, and pave the way to a "foundations-oriented introduction to non-Euclidean geometry. If the course has this character very strongly, it might be best to classify it in the category Advanced Geometry and Topology under Non-Euclidean Geometry or perhaps Other.

<u>1.9 Solid and Spherical Geometry.</u>
Much of this course is similar to "Spherical Geometry: but it will also include the necessary grounding in the geometry of points, lines, planes and spheres in 3-space.

<u>1.10 Spherical Trigonometry</u>
The relations among angles and arcs in spherical triangles created by arcs of great circles.

<u>1.11 Solid Geometry and Spherical Trigonometry.</u>
Similar to "Spherical Trigonometry" but also including the necessary preliminaries about points, lines, planes and spheres in 3-space.

<u>1.12 Surveying/Geodesy.</u>

<u>1.13 Analytic Geometry/Graphic Algebra</u>.
Coordinate geometry in the plane. Applications to algebra and analysis. Functions, graphs, limits, slope and intercept for lines, tangency, etc. Conics. If there is any extensive discussion of 3d, consider "Solid Analytic Geometry" from Category 7.

[Note to coder: "Graphic Algebra" is terminology from early in the 20th century.]

Cluster: Pre-calculus courses

<u>1.14 College Algebra/Higher Algebra.</u>
Polynomials, roots, factorization, solution of quadratic, cubic and biquadratic equations. Combinations, permutations, real and complex numbers. Mathematical induction, binomial theorem, exponential and logarithmic functions. Determinants.

<u>1.15 College Algebra and Trigonometry.</u>
Polynomials, roots, factorization, solution of quadratic, cubic and biquadratic equations. Combinations, permutations, real and complex numbers. Trigonometric functions and identities.

[Note to coder: It can be hard to distinguish this course from Higher Algebra/Theory of Equations. It is largely a matter of level (College Algebra is more elementary) and also somewhat a matter of topics. Determinants, permutations and combinations are not usually part of this course but often occur in Higher Algebra/Theory of Equations. The latter, in level and topics list, points toward linear and abstract algebra, which is why it is not in this category.]

1.16 Elementary Functions/Precalculus[3].
The nature of polynomials (maximum number of extrema, Descartes' rule, etc.), logarithmic, exponential and possibly trigonometric functions.

1.17 Comprehensive Preparation for Calculus.
Algebra, trigonometry, coordinate geometry. Differs from Elementary Functions/Precalculus in that it does more (and more elementary) algebra, and a bit more emphasis on coordinate geometry. Probably earlier in the century than 1.16.

1.18 Other

Category 2. Elementary Service Plus General Education[4] (Worksheet Abbreviation: Serv+GenEd)

Note: General Education has indistinct boundaries and may often overlap other categories in this inventory. It especially often overlaps the set of service courses which is one reason we group Service and General Education. But we do make an attempt to separate them in our clusters.

Cluster: Service of special interest for the sciences

2.1 Descriptive Geometry/Engineering Drawing.
This course is largely a how-to course for engineers about how to draw 3-d objects on paper. There is normally no prerequisite except possibly solid geometry. There is a theory to this subject and if one encounters a version of this course where the objective is to explore the theory, then that ought to go in the Advanced Geometry category.

[3] The slash indicates alternate titles or alternate wording that we consider equivalent.
[4] The terms used to define this category are late 20th century terms and may have not existed earlier. But presumably the concepts behind the terms often did. Elementary means not requiring calculus and taught at a level where it can be taken by freshmen and sophomores. (Typically this might be inferred if the course occurs near the beginning of the list of courses in the catalog.) "Service" means it is primarily intended to assist students majoring or heavily concentrating in a discipline other than mathematics, e.g., astronomy, biology, economics, and would rarely be taken by mathematics majors. A course is in the General Education category if it is elementary and meant to broaden the intellectual horizons of a broad class of students and is rarely taken by mathematics majors.

2.2 Slide Rules and Other Mechanical Devices/The Art of Computation.
Before the era of electronic calculation, slide rules, nomographs, planimeters, etc. were use to get approximate answers to a wide range of calculations. In the era before the electronic computer, every engineer had familiarity with such devices, especially a slide rule. The way in which calculations needed to be done was not obvious, hence courses of instruction, mostly by departments that served engineers. This course is a sort of forerunner of courses in scientific computation on electronic computers and also of courses in numerical analysis. Some elementary aspects of numerical analysis may be included in the course.

2.3 Boolean Algebra for Engineers.
This also appears in the algebra category. The version meant here is dedicated to circuitry, designed for electrical engineers and is not at all abstract.

Cluster: Service of special interest outside the sciences

2.4 Business Mathematics/Math of Finance.
An elementary treatment, about at the level of finite mathematics or lower.

2.5 Elementary Probability.
A first course in probability without a calculus pre-requisite.

2.6 Elementary Statistics.
A first course in statistics without a calculus pre-requisite. May contain some probability.

[Note to coder: An elementary course in biostatistics would be classified under Elementary Statistics, but with a “bio” flavor.]

2.7 Finite Mathematics/Finite Mathematics with Calculus.
Matrices, linear programming, elementary statistical and probabilistic methods. Sets and Logic. May include some calculus.

Cluster: General education

Since the 1930's there have been a number of kinds of courses meant to provide a terminal mathematics experience for non-STEM majors[5]. The next four courses show 4 important approaches for this audience. Particular courses may blend all these approaches, so we provide a fifth course when approaches have been combined or can't be determined from the course description. The courses are often more defined by the audience than by the contents. These courses, in any of their guises, are not likely to appear early in the century according to the dissertation by Michael George at Teacher's College Columbia U.

Other courses in this category, and in category 1 are sometimes allowed for general education credit at many departments.

<u>2.8 Mathematics and Culture/Mathematics and History.</u>
Mutual influences from various historical eras, or just one. Not much in the way of skill development.

<u>2.9 Structure of Mathematics/The Nature of Mathematics/Number Systems.</u>
Significant ideas in mathematics are made accessible for the many. An attempt to draw lessons about the overall enterprise of mathematics by examining serious mathematics.

<u>2.10 Mathematical Thinking.</u>
An attempt to improve the student's skills at the precise kind of thinking characteristic of mathematics, while also aiming for insight and interpretation. This course involves some skills development, often in a traditional areas such as those found in precalculus courses.

<u>2.11 Elementary Mathematical Modeling.</u>
Displaying modern mathematical applications that have only elementary mathematics at their base. Usually, the mathematics is new to the student. The kind of course that was given a big boost in For *All Practical Purposes*.

<u>2.12 Mathematics for General Education/Mathematics for the Liberal Arts</u>
A course for a broad range of students. Any blend of the approaches in the previous four courses, or any "general education" course where the approach can not be discerned.

Cluster: Miscellaneous service or general education

<u>2.13 Logic and Set Theory</u>.
Truth tables, Venn diagrams, quantified statements.

[Note to coder: one sometimes finds course with titles like Logic, Set Theory and Probability. Such a course would be handled by our policy on combination courses, as explained in the document *Classifying Courses: Standardized Courses and Categories* .]

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[5] STEM = Science, Technology, Engineering and Mathematics.

2.14 Mathematics For Women.
Elementary mathematics serving the needs of women in industry and at home. Arithmetic, algebra, geometry.

2.15 Mathematics for Agriculture Students

2. 16 Other.

Category 3: Mathematics Expressly for Teachers (Worksheet Abbreviation: ForTchrs)

These are courses that are mainly present for the benefit of prospective teachers. (In other words, if there were no math students headed for teaching, the course would probably not be there.) Many standard math courses, like linear algebra or calculus, may be recommended for teachers, but are not created largely for them. They are not included in this category. You need to judge from title and contents whether a course qualifies as a "teachers" course.

Cluster: For elementary and middle school teachers

3.1 Mathematics for Elementary School Teachers.
Content is mathematical with little or no attention to methods of teaching.

3.2 Methods of Teaching Elementary School Mathematics.
Some emphasis on methods as well as content.

3.3 Mathematics for Middle School Teachers.
Content is mathematical with little or no attention to methods of teaching.

3.4 Methods of Teaching Middle School Mathematics.
Some emphasis on methods as well as content.

Cluster: For secondary school teachers

3.5 Mathematics for Secondary School or Junior College School Teachers.
Content is mathematical with little or no attention to methods of teaching.

[Note to coders: Course which make no mention of junior college teaching, and only mention secondary school teaching, are considered in this category.]

3.6 Methods of Teaching Secondary School or Junior College Mathematics .
Some emphasis on methods as well as content.

[Note to coders: Course which make no mention of junior college teaching, and only mention secondary school teaching, are considered in this category.]

Cluster: Miscellaneous teacher's courses

3.7 Mathematics for Teachers.
A combination of "Mathematics for Elementary School Teachers", "Mathematics for Middle School Teachers" and "Mathematics for Secondary School Teachers".

3.8 Methods of Teaching.
A combination of "Methods of Teaching Elementary School Mathematics", "Methods of Teaching Middle School Mathematics" and "Methods of Teaching Secondary School Mathematics".

3.9 Topics/Seminar in the Teaching of Mathematics

3.10 Other.

Category 4: Basic Calculus Sequences (Worksheet Abbreviation: BasicCalc)

The term "basic calculus sequence" refers to a sequence of courses providing the key manipulations of differentiation and integration. This would normally include material up to and including multivariable calculus and perhaps line and surface integrals. The degree to which theory is included can vary, but courses which primarily provide the theoretical basis of calculus (theorems about continuity, differentiability, convergence, uniform convergence, etc.) and where manipulations and formulas are negligible are not included here. More advanced courses with names such as "Differential equations", "Theory of Functions", "Analysis" are not generally part of a basic calculus sequence. "Advanced Calculus" is a gray area and the course description needs to be scrutinized.

Classification is difficult in this area and use should be made of the "Flavor abbreviations" described elsewhere. There can be more than one basic calculus sequence at a single college, especially in the latter part of the 20^{th} century. There will be a mainstream sequence for math majors and many physical science students. But there may also be one, for weaker students, which integrates precalculus material with calculus; there may be a sequence for engineers; there may be a sequence for business and social science students, etc. We provide for 7 named sequences and the ubiquitous "Other" sequence.

We use flavors to give brief clues about the nature of the individual courses in a sequence. For example "fl: ag" indicates that the course included analytic geometry.

Each sequence is also a cluster.

Cluster: Mainstream basic calculus sequence

These courses may be with or without analytic geometry, the status to be indicated by the flavor abbreviation "ag". This is a sequence recommended for mathematics majors and often many science and engineering students. In cases where there is only one calculus sequence suitable for mathematics majors, that is, by default, the mainstream basic sequence. In cases where there are two sequences, one of them faster or "honors", then the "mainstream basic sequence" is the one which is not faster and not "honors". We use the name "Accelerated mainstream basic calculus" for a faster or "honors" sequence (see below). The mainstream basic calculus sequence always presents a good deal of the manipulative principles of differentiation, integration, series and multivariable calculus, possibly including line and surface integrals. Some theory may be presented as well, but this varies from department to department and time to time. In cases we are aware of this sequence consists of more than one term, but depending on the department, the pace of the courses, number of hours in the courses, and whether semesters or quarters are in use, the number of terms can be anywhere from 2 to 5. Courses entitled "Advanced Calculus" may be part of this sequence if they are substantially manipulative – see the catalog description and our later discussion of advanced calculus. A brief introduction to advanced topics such as differential equations or complex variables might occur in this sequence. No prior training in calculus is necessary for taking this course.

4.1 Mainstream Calculus Term 1
4.2 Mainstream Calculus Term 2
4.3 Mainstream Calculus Term 3
4.4 Mainstream Calculus Term 4
4.5 Mainstream Calculus Term 5

Cluster: Calculus with precalculus

For students with inadequate preparation in algebra, trigonometry, etc. This sequence is typically meant to feed students into some course in the mainstream sequence (which is why we provide only 3 courses in this sequence.) In some cases there is only a single course in this sequence.

4.6 Calculus With Precalculus Term 1
4.7 Calculus With Precalculus Term 2
4.8 Calculus With Precalculus Term 3

Cluster: Accelerated or honors mainstream basic calculus sequence

4.9 Accelerated/Honors Calculus Term 1
4.10 Accelerated/Honors Calculus Term 2
4.11 Accelerated/Honors Calculus Term 3

Cluster: Basic calculus sequence for students of science, engineering and technology.

Similar to the mainstream sequence but with more practical concerns (more applications, less theory) and possibly a sequencing of topics that harmonizes better with what students will be learning in science and engineering and technology courses. We provide up to 5 terms.

4.12 Calculus for Sci/Eng/Tech Term 1
4.13 Calculus for Sci/Eng/Tech Term 2
4.14 Calculus for Sci/Eng/Tech Term 3
4.15 Calculus for Sci/Eng/Tech Term 4
4.16 Calculus for Sci/Eng/Tech Term 5

Cluster: Briefer calculus sequence for students outside the sciences.

Few if any transcendental functions or multivariable calculus. Up to 3 terms. (These courses are frequently called by titles such as *Calculus for Business*, *Calculus for Life Sciences* or *Calculus for Business and Life Sciences*.)

4.17 Briefer Calculus Term 1
4.18 Briefer Calculus Term 2
4.19 Briefer Calculus Term 3

Cluster: Calculus with theory.

This typically assumes students have a knowledge of basic calculus. This arguably belongs in the *Analysis Following Basic Calculus* category, except that this sequence is available as the first mathematics a student takes (for unusually well-prepared students.)

4.20 Calculus With Theory Term 1
4.21 Calculus With Theory Term 2
4.22 Calculus With Theory Term 3
4.23 Calculus With Theory Term 4
4.24 Calculus With Theory Term 5

Cluster: Calculus for students with prior exposure to calculus.

Some colleges (e.g., MIT, Reed) have a special sequence for those who bring transfer credit in calculus or advanced placement or knowledge gained other ways. At MIT, this sequence seem to be based on the idea that the knowledge gained elsewhere might not really be up to local standards. Typically, this sequence feeds into the mainstream sequence or to whatever might follow basic calculus (e.g. differential equations.) It differs from the “With Theory” cluster in that it does not have unusual emphasis on theory.

4.25 Calculus For Those With Prior Exposure Term 1
4.26 Calculus For Those With Prior Exposure Term 2
4.27 Calculus For Those With Prior Exposure Term 3

Cluster: Other calculus sequence.

4.28 Other Calculus Term 1
4.29 Other Calculus Term 2
4.30 Other Calculus Term 3
4.31 Other Calculus Term 4
4.32 Other Calculus Term 5

Cluster: Miscellaneous calculus

4.33 Vector Analysis/Vector Calculus.
The study of vector algebra and vector fields. The differentiation and integration of vector fields. The gradient theorem, Green’s theorem, Stoke’s theorem, the divergence theorem. Applications to electromagnetic fields, gravitational fields and fluid flow. Tensors.

[Note to coder: Much of the material in this course made its way into the mainstream basic calculus sequence, often under the course title "Multivariable Calculus", but sometimes distributed throughout the sequence. To determine whether a course found in a catalog should be classified as *Vector Analysis/Vector Calculus* or is part of the calculus sequence, or as *Advanced Calculus* use these guidelines: if the course contains line and surface integrals (Theorems of Green and Stoke, the divergence theorem) and contains only material pertaining to vector fields (e.g., no power series, techniques of integration, ordinary differential equations) then classify it as vector analysis. If line and surface integrals are present but with other calculus material which is not elementary (e.g., complex functions), perhaps Advanced Calculus would be better. If it contains line and surface integrals but also elementary calculus material, such as power series, see if it can fit into a basic calculus sequence. Heavy emphasis on tensors might indicate that the course is best classified as *Tensor Analysis* in Category 9.]

<u>4.34 Other Basic Calculus</u>
Topics and applications, perhaps for special categories of students. E.g., an overview for a broad array of students, serving rather like a general education course.

Category 5: Analysis Following Basic Calculus (Worksheet Abbreviation: Anal)

Cluster: Extended basic calculus

<u>5.1 Curve Tracing</u>.
Tracing by points, symmetry. Orders of small quantities, forms of parabolic curves near the origin or at an infinite distance. Forms of curves in the neighborhood of the origin, multiple points. Forms of branches whose tangents at the origin are the coordinate axes. Asymptotes, including curvilinear asymptotes. The analytical triangle. Singular points. Systematic tracing of curves, repeating curves. Determination of the equation of a given curve.

<u>5.2 Infinite Series/Power Series.</u>
An extended version of the shorter treatment often presented (circa 2009) as part of a semester of calculus. Convergence tests, power series, infinite products, operations with series. Inversion of power series. Theorems of Abel, Dini and Pringsheim. Abel's and Dirichlet's tests. Asymptotic formulas for divergent series.

Cluster: Advanced calculus

<u>5.3 Advanced Calculus with No Course Description</u>

The reason for this entry in our inventory is that Advanced Calculus has meant many things (see below), many of which are classified under other standardized courses in this inventory. Without a course description (a situation that occurs often in the early part of the century) we can't tell whether to map a given catalog entry entitled *Advanced Calculus* to our inventory entry *Real Analysis* or to *Complex Analysis* or what. In such a case we use the standardized course described here. Of course, this is little more than an expression of ignorance. If there is a textbook mentioned we should cite it (e.g. "tx: Kaplan's *Advanced Calculus*".) We also indicate the catalog title by "ti: *Advanced Calculus*".

Examples of course content we have found in courses entitled *Advanced Calculus*:

1. Lots of what today would be regarded as special topics about single variable derivatives and integrals (techniques of integration not recently taught, evolutes, involutes, osculations, types of tangency). Such a course is basically an extension of today's Calculus 1 and Calculus 2. The examples we have seen were mostly early in the century. We map this to one of the entries in the basic calculus sequence.

2. Multivariable topics, such as the implicit function theorem, line and surface integrals (first showing up around 1935). We map this to some entry in the calculus sequence (the last course of the sequence) or maybe to *Vector Analysis.*

3. Introductory real analysis, with theorems about limits, continuity, etc. The first pure theory course in analysis. We map this to *Real Analysis*.

4. Topics specially directed at engineers and physicists (special functions, Fourier series and transforms) We map this to *Engineering Mathematics/Methods Of Applied Mathematics, or Mathematical Physics*

5. Ordinary differential equations. The obvious mapping.

6. Partial differential equations. The obvious mapping.

7. Complex variables. The obvious mapping.

8. Power series. The obvious mapping.

9. Finally, there may be a hard-to-classify mix of topics chosen from the areas previously listed here. For this we have the following standardized course.

5.4 Advanced Calculus: Mixed Topics.

A mix of topics, not easily classified any of the previous ways.

Cluster: Differential equations and related courses

5.5 Differential Equations.
The harmonic equation (mx" = -kx) and linear ordinary differential equations of the first and second order. Existence theorems, methods of solution, algebraic and geometric. Sturm-Liouville systems, etc. special solutions. The theory of envelopes. May include linear algebra.

5.6 Dynamical Systems.
Non-linear differential equations. Iterated function systems, bifurcations, periodicity, attractors, limit cycles, chaos. Fractals.

5.7 Numerical Analysis.
Numerical solution of differential equations (such as the Runge-Kutta method.) Approximation to solutions of integral equations. Fast fourier transforms and arithmetic with arbitrary precision. Finding eigenvalues and other problems in linear algebra. Finite element analysis.

Cluster: The theory of functions

5.8 Real Analysis.
Possibly including some instances of Advanced Calculus.

The theory behind calculus. Differentiability, continuity, integrability. Proofs using limits. Elements of a topological viewpoint. Advanced or accelerated versions might include measure and function spaces (Hilbert and Banach spaces) and theories of integration. This course has little or none of the formulas and manipulations of basic calculus. Note that not every course with title that includes “Advanced Calculus” is a reasonable match for the course being described here. See categories 4 and 9 for other possibilities.

5.9 Complex Analysis.
Possibly including some instances of Advanced Calculus.

Complex numbers, Cauchy's theorem and complex differentiation and integration. Analytic functions, analytic continuation, the Riemann mapping theorem, conformal mapping, etc. Advanced versions might include: meromorphic functions, harmonic and subharmonic functions. Theorems of: Rouche, Hurwitz, Runge, Weierstrass, Hadamard, Mittag-Leffler, etc.

5.10 Real and Complex Analysis.
Basic ideas of real analysis and basic ideas of complex analysis.

[Note to classifiers: in the early 20th century, textbooks (and probably courses) such as Frank Morley and James Harkness’ “A Treatise on the Theory of Functions” combined real and complex analysis, but with the real variable material being brief and only in support of the more ambitious complex variable material. Such a course may be more appropriately classified as "Complex Variables". But it may not be possible to tell from a catalog description if the real variable material is really slight enough to justify the classification "Complex Variables". Hence this combined standardized course is made available.]

Cluster: Advanced analysis

<u>5.11 Measure and Integration.</u>
Measure spaces, Lebesgue measure and the Lebesgue integral. Product measures, Lp spaces, integral representations of linear functionals. Probability spaces.

<u>5.12 Introduction to Functional Analysis/Linear Operators.</u>
Hilbert, normed and Banach spaces, linear operators, dual spaces.
Representation theorems. Theorems of Riesz-Fischer, Hahn-Banach.

<u>5.13 Calculus of Variations.</u>
The calculus of variations is concerned with finding optimal solutions (shapes, functions, etc.) where optimality is measured by minimizing a functional (usually an integral involving the unknown functions) possibly with constraints. This course is an introduction to the classic ideas and techniques of the calculus of variations, such as the associated Euler-Lagrange equation, the Beltrami identity, and Dirichhlet's Principle . There may be applications such as: Fermat' Principle, isoperimetric problems, the Hamilton-Jacobi differential equation, eigenvalue-eigenfunction problems for the vibrating string.

<u>5.14 Integral Equations.</u>
Equations of Volterra and Fredholm. The Hilbert-Schmidt theorem. The Wiener-Hopf method. Singular integral equations of Cauchy type. Applications to solid mechanics, quantum mechanics, acoustics.

<u>5.15 Harmonic Analysis and Wavelets.</u> Fourier series on the circle, Fourier transforms on the line and in space. The Discrete Wavelet Transform, the Fast Wavelet Transform and filter-bank representation of wavelets. Applications to such areas as fingerprint representation and algorithmic recognition.

Cluster: Miscellaneous analysis

<u>5.16 Topics/Seminar in Analysis.</u>

<u>5.17 Other.</u>

Category 6: Higher and Abstract Algebra, Linear Algebra and Number Theory (Worksheet Abbreviation: Alg+NumTh)

Cluster: Concrete studies in algebra

6.1 Quaternions.
Hamilton products and the algebra of quaternions. Conjugation, the norm, and division. Quaternions and the geometry of three-space. Connection to mechanics.

6. 2 Higher Algebra/Theory of Equations.
Binomial and multinomial theorems, permutations and combinations, infinite series, continued fractions. Descartes' Rule. Discriminant of an equation, polynomials, roots, approximation methods. Linear systems of equations.

6.3 Determinants.
Effect of elementary row and column operations. Methods of calculating a determinant. Application to solving linear systems of equation. Connection to linear transformations. Special determinants such as circulants.

6.4 Matrix Theory.
Computational methods with matrices. Diagonalization. Linear equations (perhaps an introduction to linear programming, but an emphasis on the practical applications of matrix theory.) Linear transformations may be studied (and perhaps invariants of particular types as well) but concretely, as given by linear equations in n variables. Little or nothing about vector spaces or the abstract view of linear transformations.

6.5 Boolean Algebra.
Algebra of sets, propositional calculus, the algebraic and axiomatic viewpoint. Applications to switching circuits.

Cluster: Linear algebra

6.6 Linear Algebra.
Vector spaces, linear transformation. Linear independence and bases, matrix representations. Equivalence and similarity. Sets of linear transformations. Modules and other generalizations.

6.7 Quantics.

Quantics are homogeneous polynomials in any number of variables. The terminology dates from the 19th century – probably Cayley or Sylvester. Invariants of quantics is likely to be a main topic in the course.

Cluster: Abstract structures

6.8 Abstract/Modern Algebra without Linear Algebra.
Groups, rings, fields, vector spaces. Definitions and fundamental theorems. Homomorphisms and isomorphisms. Polynomials and field extensions.

6.9 Abstract/Modern Algebra with Linear Algebra.
Combination of Linear Algebra and Abstract/Modern Algebra without Linear Algebra.

6.10 Group Theory
Groups and subgroups. Special groups such as cyclic and permutation groups, symmetry groups, free groups. Normal subgroups, cosets, homomorhisms, Lagrange's theorem. Representations by matrices or by generators and relations. Cayley graphs.

6.11 Group Representations
Representing the elements of a group by invertible matrices under matrix multiplication. Irreducible representations. Modular representations. Applications to physics, chemistry.

6.12 Galois Theory.
The relation between roots and coefficients of a polynomial: elementary symmetric functions; complex roots of unity; and solutions by radicals of cubic and quartic equations. The characteristic of a field and the prime subfield. Factorisation and ideal theory in polynomial rings. The structure of a primitive field extension. Field extensions and characterisation of finite normal extensions as splitting fields. The structure and construction of finite fields. Counting field homomorphisms; the Galois group and the Galois correspondence. Radical field extensions. Soluble groups and solubility by radicals of equations.

6.13 Finite Fields/Coding Theory
Structure of finite fields and applictions to coding theory and number theory. Equations over finite fields, including Gauss and Jacobi sums, reciprocity theorems and special cases of the zeta function. Cyclotomy with applications to difference sets. Finite Fourier analysis.

Cluster: Number theory

6.14 Number Theory.
Definition and elementary properties of integers. Induction, factorization, congruences, residues, primitive roots, indices, etc.

6.15 Algebraic Number Theory
Algebraic number fields. Factorization, ideals, field extensions. Class numbers, Dirichlet's theorem, quadratic and cyclotomic fields.

6.16 Analytic Number Theory.
Dirichlet L-series, the totient function, the use of zeta functions and L-functions to prove distribution results concerning prime numbers (e.g., the prime number theorem in arithmetic progressions). Additive number theory including Goldbach's conjecture and Waring's problem.

Cluster: Miscellaneous algebra

6.17 Topics/Seminar in Algebra

6.18 Other.

Category 7: Advanced[6] Geometry and Topology (Worksheet Abbreviation: AdvGeo+Top)

Cluster: Advanced euclidean geometry

7.1 Advanced Synthetic Euclidean Geometry/College Geometry/Ruler and Compass Constructions/.
Theorems of Ceva, Menelaus. The nine-point circle, the Simpson Line, Napolean's theorem. Special points in a triangle. Ruler and compass constructions.

7.2 Conics/Advanced Analytic Geometry/Modern Analytic Geometry.
What distinguishes this course from "Analytic Geometry/Graphic Algebra" in category 1 is one or more of the following: the phrase "modern methods", extensive treatment of conics, use of homogeneous coordinates or other kinds of alternative coordinate systems (Plücker, Grassmann), points and lines at infinity (the projective point of view), Plücker equations, significant attention to transformations and their invariants. If there is

[6] A geometry course is advanced if it is not one of: Plane Geometry (as in high school), Plane Analytic Geometry, Solid Geometry, Trigonometry, Spherical Geometry, Spherical Trigonometry, Descriptive Geometry. Solid Analytic Geometry is advanced. Often "advanced" coincides with "upper division" but not always. At Berkeley in 1925 Projective Geometry was lower division. Solid Analytic Geometry was a freshman course at City College of New York in 1906.

extensive treatment of 3d then the course might best be classed as "Solid Analytic Geometry".

[Note to coder: The article cited below includes: "In many American universities the courses in Modern Geometry have been during the past two decades courses in advanced analytic geometry. The text followed was either Salmon's conic sections, Smith's conic sections or some other text emphasizing the special properties of conic sections with some attention to homogeneous coordinates, abridged notation, etc., or else it was a text such as Miss Scott on *Modern Analytic Geometry*."]

<u>7.3 Solid Analytic Geometry.</u>
Lines and planes in space. Tangent planes. Quadric surfaces. Classification and invariants of quadrics. Matrix methods may be used.

Cluster: Projective and non-Euclidean geometry

<u>7.4 Projective Geometry.</u>
Theorems of Pappus, Desargues. Projections, collineations, projective transformations. Projective equivalence of conics. Pascal's theorem on conics. Cross ratio. This could be synthetic or coordinate based. If the latter, homogeneous coordinates will be included.

<u>7.5 Non-Euclidean Geometry/Theories of Geometry</u>
Systems of geometry that deny Euclid's parallel postulate: hyperbolic, spherical and elliptic geometries. Questions of consistency. The Klein or Poincare models. The course may include comparisons among geometries. Projective geometry may be used as an aid in a unified and comparative point of view.

<u>7.6 Geometry and Group Theory</u>.
Transformations of various kinds in various geometries (Euclidean, non-Euclidean, projective) and groups of invariants. The Erlangen Program.

Cluster: Geometry with other mathematical machinery

<u>7.7 Point Set Topology</u>.
Sets, cardinal numbers, functions, limits, continuity. Elementary point set topology. Metric spaces. Open and closed sets.

<u>7.8 Algebraic Topology/Combinatorial Topology</u>.
Simplexes, complexes, Betti numbers, homotopy groups, homology groups. Coefficient theorems. Applications to geometry and analysis.

7.9 Algebraic Geometry/Algebraic Functions.
Algebraic varieties over various fields. Simple and singular points of plane curves. The resultant. Bezout's theorem. Abelian varieties, Abelian functions.

7.10 Differential Geometry.
Calculus methods applied to curves and surfaces. Tangent vectors and tangent planes. Various concepts of curvature, torsion.

Cluster: Miscellaneous geometry

7.11 Convex Sets.
Intersections of convex sets. Theorems of Helly, Radon, Caratheodory. Support hyperplanes. Convex polyhedra. Curves of constant breadth. Convex functions and relation to inequalities.

7.12 Modern Geometry With No Course Description.

Early in the 20th century, titles often appeared with no course description. In many cases, the title is fairly descriptive. But "Modern Geometry" is not such an easy-to-understand case. From the present vantage we can imagine many interpretations for "modern". Fortunately a bit of surfing turned up this Monthly article: A Course in Geometry for College Juniors and Seniors, by J. N. Van Der Vries *The American Mathematical Monthly*, Vol. 24, No. 1 (Jan., 1917), pp. 21-23 1917. Therein we find:

"The term Modern Geometry seems to have grown up in our mathematical nomenclature for the purpose of distinguishing a college course in geometry under this name from the more elementary course in the secondary school, this latter being the elementary geometry of the Greeks, practically unchanged."

[The author goes on to say there is a bit of a trend to making Modern Geometry mean synthetic projective geometry. This illustrates his earlier remark that there is little consensus on what Modern Geometry means. He also claims that Modern Analytic Geometry and Modern Synthetic Geometry have not much consensus either.]

If we see course with the title Modern Geometry, but where there is a course description, we should classify it as one of the other courses in this category.

7.13 Higher Geometry.
This vague term may be encountered as a course title early in the century. If there is a course description, see if the course can be classified as one of the other courses in this category which has a more informative title. Otherwise, for example if the course is a catchall with topics such as contact transformations, inversion, a bit of projective geometry, or if no topics are mentioned, use this title.

7.14 Survey Of Geometry/Modern Geometry.
A variety of topics showing the breadth of geometric ideas. Topics might include finite geometries, different axiom systems, tilings, symmetry, transformation geometry.

7.15 Topics/Seminar in Geometry Or Topology

7.16 Other.

Category 8: Foundations
(Worksheet Abbreviation: Found)

Cluster: Logic and set theory

8.1 Mathematical Logic/Symbolic Logic
Propositional Calculus (Truth tables.) The first order predicate calculus. Perhaps some advanced topics such as recursive functions, decidability (Gödel's Theorem), etc. Applications of logic to mathematics or switching circuits.

[This course is aimed at math majors and needs to be distinguished from Boolean Algebra courses (see the Elementary Service+ General Education category and the Algebra+Number Theory category) aimed at electrical engineers (and convey practical skills in analyzing or simplifying "switching circuits" or digital circuits) or at abstract algebra issues in which one loses sight of the fact that our symbols stand for statements, that quantification may occur, etc. Tipoffs may include: Lack of high end math, lots of mention of circuit issues (e.g., minimizing Boolean expressions, Karnaugh maps), less than 3 semester hours, title, course number (Boolean Algebra for engineers was often a lower division course).]

8.2 Set Theory .
Operations on sets. Higher cardinal numbers, ordinal numbers.

8.3 Mathematical Logic and Set Theory.
Propositional Calculus (Truth tables. The first order predicate calculus. Applications of logic to mathematics. Decidability. Operations on sets. Higher cardinal numbers, ordinal numbers.

Cluster: Miscellaneous foundations

8.4 Fundamentals of Mathematics/Foundations of Arithmetic.

An introduction to the foundations of mathematics. Logic, sets, functions, equivalence and order relations, various axiomatic systems. Algebraic systems. Peano postulates for the natural numbers, construction of various number systems, including the real numbers.

8.5 Introduction to Proofs/Transition to Higher Mathematics.
Proof techniques: induction, direct and indirect proofs. Emphasis on the clarity, precision and correctness in expressing mathematical ideas.

8.6 Topics/Seminar in Foundations

8.7 Other.

Category 9: Advanced Applied Courses (Worksheet Abbreviation: AdvAp)[7]

By "applied courses" we usually mean those whose *titles* announce that modeling, applicability, or an applied point of view, or certain particular applications are central to the design of the course. Advanced means that these are typically taken by juniors or seniors who have been through a good deal of a calculus sequence. In particular they are not service courses.

Finite mathematics, which typically contains applications, is not in this category because it is a service course, and not advanced. Calculus is not advanced. "Applied Linear Algebra" is in the this category. Subjective distinctions obviously.

Cluster: Mathematical tools used mainly in physical science/engineering

9.1 Engineering Mathematics/Methods Of Applied Mathematics.
Possibly including some instances of Advanced Calculus.

A selection of many topics, each covered lightly ("how-to" emphasized more than proofs.) Vector calculus, linear algebra, complex variable, numerical methods, optimization involving graphs (networks), differential equations. Applications of analysis and differential equations to problems of physics and chemistry. The Laplace and Fourier transforms, Fourier series and expansions.

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[7] Arguably, much or all of *Category 11: Advanced Probability and Statistics With Mathematics Designations* could be included in the present category, but it seemed an undeserved denigration for probability and statistics not to appear at the category level, or to appear in hollowed out form. However, any study of applications in the curriculum should take category 11 into account. Similar remarks apply to category 12 (*Computer Science*.)

[Note to coders: In rare cases this might be called “Operational Mathematics” after the title of a well-known book by Churchill.]

9.2 Mathematical Physics.
Possibly including some instances of Advanced Calculus.

Similar to Engineering Mathematics, but more sophistication in topics and approach. Complex variables, differential and integral equations, boundary value problems, transforms, Sturm-Liouville theory, calculus of variations. Directed at Math or Physics majors more than engineers.

9.3 Tensor Analysis/Vector and Tensor Analysis
Index notation and the summation convention. Contravariant and covariant vectors. Symmetry and Skew-symmetry. Multilinear algebra. Cartesian and General tensors. Line elements. Geodesics, curvature tensors. Differential Tensor Calculus. Riemannian geometry. Applications may include: rheology, relativity, continuum mechanics, elasticity.

9.4 Partial Differential Equations /Boundary Value Problems.
Possibly including some instances of Advanced Calculus

The method of characteristics. Existence theorems and methods of solution. Fourier series and transforms. The Cauchy-Kovalevsky Theorem. The Dirichlet Problem and Laplace's equation, Bessel functions, orthogonal polynomials. Sturm-Liouville problems. Special equations such as the Hamilton-Jacobi equation. Eigenfunctions and classification of PDE’s.

[Note to classifiers: the flavor abbreviation “Ap” should be used if there seems to be significant mention of applications such as heat transmission, waves, vibrating membranes.]

9.5 Special Functions/Elliptic Functions/Abelian Functions.
Elliptic functions, Bessel functions, orthogonal functions, Hermite-Rodriguez functions , Legendre polynomials, etc.

[Notes to coder:
1. any course named for a particular family of functions should be considered an example of this standardized course.]

2. Note the similarity of the topic list to topics found in other courses mentioned here. The coder should be guided by the title even more than usually.]

9.6 Laplace Transforms / Fourier Series/Theory of Transforms.
Examples of partial differential equations describing physical systems, Fourier series,

basic separation of variables, Sturm Liouville Theory, separation of variables with more complicated boundary conditions or sources, Fourier transforms, similarity methods, power series solutions of ordinary differential equations, separation of variables in spherical coordinates, Legendre polynomials and Fourier-Legendre expansions, regular singular points and the method of Frobenius, separation of variables in cylindrical coordinates, Bessel functions and Fourier-Bessel expansions. Linear systems of differential equations, convolution, elementary development of the delta function, and the use of transforms to solve systems.

[Note to coder: these course can be theoretical or with emphasis on applications. Use flavors if you can detect a significant emphasis.]

<u>9.7 Generalized Functions (Distributions).</u>
Test functions, functionals, continuity, distribution derivatives, Fourier Transforms, convolution, Green's function, application to ordinary and partial differential equations.

<u>9.8 Control Theory</u>
Controllability, observability, dynamic programming, maximum principle, optimal control, Lyapunov stability.

Cluster: Modeling branches of physical science/engineering

<u>9.9 Mechanics/Analytical Mechanics/Rational Mechanics/Analytical Statics.</u> A course in classical mechanics from the mathematical point of view. How forces and torques acting on bodies or systems of bodies create velocities and accelerations or leave the bodies(s) in equilibrium (Newton's laws.) Basic kinematics. Harmonic motion. Work, energy, momentum. Conservation laws. Reference frames. Lagrangian mechanics. Elasticity. Fluid Mechanics.

Rational mechanics describes an especially rigorous point of view for the same material in which physically obvious assertions are not allowed in arguments unless they have been specified in advance as axioms or proved as theorems.
Statics is the half of mechanics dealing with objects in equilibrium.

<u>9.10 Celestial Mechanics.</u>
The forces and resulting motions involving the heavenly bodies.

<u>9.11 Signal Processing</u>

Fourier series, sampling and aliasing, time and frequency analysis, filter design, z-transform, spectrum analysis, Discrete and Fast Fourier Transform, analog/digital conversion, wavelets, multirate signal processing.

9.12 Potential Theory
Properties of functions that satisfy Laplace's equation. Newtonian and vector potential, differential operators, problems related to Maxwell's equation, harmonic functions, Green functions, subharmonic functions, kernels.

9.13 Elasticity
Static and dynamic problems including linear elastic waves, Hooke's law, beam theory analysis of stress and strain, the constitutive equations, biharmonic equation, two dimensional problems, problems of prismatic bars, variational methods and energy principles, Beltrami-Michell equation.

9.14 Thermodynamics
First and second laws of thermodynamics; thermodynamic properties of gases, vapors, and gas-vapor mixtures; energy-systems analysis including power cycles, refrigeration cycles and air-conditioning processes. Introduction to thermodynamics of reacting mixtures. The Gibbs equation.

9.15 Hydrodynamics/Fluid Mechanics/Fluid Dynamics.
The motions resulting from forces acting on air or liquids. The Navier-Stokes equations. Turbulence, laminar flow.

[Note to coder: in the 19^{th} century hydrodynamics was synonymous with fluid dynamics. With the development of meteorology and of aircraft it became useful to use hydrodynamics for the case where the fluid was water and fluid dynamics or when the "fluid" was air.]

9.16 Exterior Ballistics
The path of projectiles such as bullets. The back curve, the ballistic coefficient, the bore centerline, the critical zone.

Cluster: Mathematical tools often applicable outside physical science/engineering

9.17 Applied Linear Algebra. . Similar to standard linear algebra but applications such as to Leontieff input-output models, linear programming, differential equations, computer graphics, geometric transformations, population modeling, electrical circuits., etc.

9.18 Applied Abstract Algebra/Algebraic Coding Theory. The applications would typically involve codes for error correction such as Hamming codes, or data compression methods such as Huffman codes, although other applications might appear. Some of courses in this category may have no algebra prerequisite and take on the task of providing basic instruction in an area of algebra, e.g. basic algebraic structures such as groups, rings and fields (especially finite fields), or vector spaces, and then add applications as time permits. Alternatively, the course might require at least one prerequisite in either abstract algebra, linear algebra or number theory and then have more intense work on the applications.

[Note to coder: We consider these two forms of the course (with or without algebra prerequisites), to be different instances of the same course, distinguished in our worksheet by the algebra or number theory prerequisite or lack of it.]

Cluster: Modeling areas outside physical science/engineering, operations research, actuarial science

9.19 Mathematical Economics.
Monopoly, oligopoly, competition, taxation, utility, economic dynamics, general equilibrium, stability of equilibrium prices, welfare economics, growth and discounting, game theory, statistics, and econometrics (statistical methods in economics), Leontieff's input-output analysis.

9.20 Mathematical Biology.
Many possibilities for topics: genetics, molecular biology (DNA, RNA), epidemiology, ecology, etc. Mathematical methods could include ordinary and partial differential equations, difference equations, probability, combinatorics, etc.

9.21 Bioinformatics
Biological database queries (BLAST); sequence alignment; edit distance; gene finding; hidden Markov chains; phylogenetics; protein structure prediction.

[Note to coder: if the course concentrates on a single topic or a single mathematical method, add a flavor.]

9.22 Number Theory and Cryptology.
Cryptography (making codes), cryptanalysis (breaking codes). Block ciphers, modes of operation, hash functions, digital signatures. Advanced Encryption Standard, and elliptic curve cryptography. Symmetric (also known as private key) and asymmetric (also known as public key) encryption, RSA, the discrete logarithm problem, public-key infrastructure, key distribution, and various applications.

9.23 Cryptology.
Similar to *Number Theory and Cryptology* except that cryptology in all of its aspects (not just number theory) is under discussion. Thus combinatorics could arise, non-mathematical topics such as one-time pads would be discussed, computational issues (including quantum computing perhaps.) But note, it has to be a course with MTH designation.

Cluster: Actuarial science

Actuarial exams have been given for about 100 years and have changed their nature over time. At the outset, the first one concerned English grammar. The latest era starts Jan 1, 2000. Most of the courses we list below are largely devoted to the individual actuarial exams either in the era starting in 2000 or the era just before. But note that we maintain a distinction between a course presenting new material and a course devoted to drill and preparation for an exam.

From surfing three actuarial programs (
Purdue – http://www.math.purdue.edu/academic/actuary,
Stony Brook - http://www.ams.sunysb.edu/undergraduate/actuarialtraining.shtml,
- and U. Texas at Austin – http://www.ma.utexas.edu/dev/actuarial/)
and the Society of Actuaries website (**http://www.soa.org/education/exam-req/edu-asa-req.aspx**) it seems reasonable to assume that:
1. Courses in actuarial science parallel the syllabi for the exams fairly strongly.
2. In some cases, more than one course may be given over to the topics we package into one course. We can handle this my making multiple entries in the cell for the one course we created. We call this multiple "instances" of a course. (Two instances of a course might be alternatives – one could take one or the other – or members of a sequence. The sequence form of multiple instances is most likely here.)
3. Some of the courses we describe below might be in a statistics department or a business school. Those seeking a full understanding of actuarial science need to remember that what we present here is the mathematics department's role in actuarial science.
4. In at least one department we found a "gentle introduction" to actuarial science that was not sufficient preparation for any exam, and only 2 credits. This would be an example of our "Financial and Actuarial Mathematics."

Courses likely to be found in both the pre-2000 and post-2000 era.

<u>9.24 Drill and Preparation for an Actuarial Exam.</u>
Such exams might be devoted to: Calculus; Calculus and Linear Algebra; Probability; Statistics and Finite Differences (all in the pre-2000 era), Theory of Interest (both eras), Probability and Statistics (post-2000 era.) The coder should enter the name or code for the exam at the end of the cell entry.

<u>9.25 Elementary Financial and Actuarial Mathematics.</u>
Basics of the theory of interest. Issues concerning mortality: Annuities, life tables, expectation of life. Financial aspects of insurance and pensions. Introduction to statistical methods in finance. This course combines features of "Financial Mathematics" and "Actuarial Models For Life Contingencies" and is an introductory course. But the level of mathematics is higher and might include calculus.

[Note to coders: This is a more advanced treatment of financial mathematics than in "Business Mathematics/Math of Finance" in the "Elementary Service Plus General

Education" category , in part because it includes some actuarial matters (e.g., mortality issues), and in part due to the mathematical level. It might have a calculus prerequisite.

9.26 Finite Differences, Interpolation and Numerical Analysis for Actuaries
Ordinary differences and related operators, dividend differences; polynomial interpolation, numerical differentiation, osculatory interpolation, , approximate int4gration; difference equations. Applications to actuarial issues.

9.27. Theory Of Interest
Simple and compound interest and annuities, bonds and loans, portfolios and immunization. How those fundamental concepts are applied in calculating present and accumulated values for various streams of cash flows as a basis for future use in: reserving, valuation, pricing, asset/liability management, investment income, capital budgeting, and valuing contingent cash flows. Assumes basic knowledge of calculus and probability.
(As in pre-2000 actuarial exam #140. But covers only a portion of the post-2000 Exam FM.)

9.28 Actuarial Modeling.
This is a large subject, to which many catalog courses may be mapped. Course titles might include: *Credibility Theory and Loss Distributions*, *Single and Multiple Event Losses*, *Life Contingencies and Contingent Payments*. Topics might include: Markov chain models, present-value-of-benefit random variables and their expectations, variances. Poisson processes, estimation of parameters for a model. The effects of deductibles, limits, coinsurance; calculation of loss elimination ratios; evaluation of the effects of inflation on losses. Risk measures. Failure time and loss distributions (Kaplan-Meier estimator, Nelson-Åalen estimator, Kernel density estimators, the Kolmogorov-Smirnov test, the Anderson-Darling test, Chi-square goodness-of-fit test, the likelihood ratio test, the Schwarz-Bayesian criterion.) Construction and selection of parametric models. Simulation including the bootstrap and inversion methods.

The life contingencies materials is aimed at portion MLC of post-2000-era Exam M. Other material aimed at Post-2000 era Society of Actuaries Exam C (Casualty Actuary Society Exam #4). In the pre-2000 era exams targeted would be: Casualty Actuary Exam 4B, and Society of Actuaries Exam 150.

A thorough knowledge of calculus, probability, and mathematical statistics is assumed.

9.29 Other Actuarial.
The likely possibilities here are: reading courses (conference courses), internships, seminars.

Courses likely to be found mostly in the pre-2000 era.

9.30 Statistical Methods for Actuaries.
Regression, analysis of variance, time series analysis. Aimed at Society of Actuaries Exam 120.

9.31 Operations Research for Actuaries.
Linear programming, project scheduling, dynamic programming, queueing theory, decision theory, and simulation. Aimed at Society of Actuaries Exam 130.

Courses likely to be found mostly in the post-2000 era.

9.32 Financial Modeling.
An elementary course in this subject might combine the topics in "Theory of Interest" with financial economics including: derivatives, options, hedging and arbitrage, forwards, futures, swaps. This would be tailored to post-2000 Exam FM of the Society of Actuaries (also Casualty Actuary Society Exam 2) in Financial Mathematics. Assumes a basic knowledge of calculus and an introductory knowledge of probability.

A more advanced course might cover the Vasicek and Cox-Ingersoll-Ross bond price models the Black-Derman-Toy binomial model, the valuation of derivative securities (Black-Scholes equation, diffusion processes, Ito's lemma), simulation of security prices (e.g., Monte-Carlo techniques), risk-management techniques. A thorough knowledge of calculus, probability, and interest theory is assumed. Keyed to post-2000 Exam M, segment MFE (Models of Financial Economics) of the Society of Actuaries (also Exam 3F of the Casualty Actuary Society.)

Cluster: Operations research

9.33 Operations Research/Management Science/Industrial Engineering.
Optimizing activities in organizations of all types. This course will probably include linear programming but it has a broader selection of topics which might include: decision theory, inventory analysis, network flows, queues, game theory, scheduling. The course might even be restricted to one of the aforementioned. Models can be deterministic or stochastic.

[Note to coder: The last two versions of the course title are the titles usually used for this kind of material in business schools or engineering schools respectively. These titles will probably not be found in a mathematics department, but who knows?]

9.34 Linear Programming/Mathematical Programming/Optimization.
Optimization over feasible regions. Linear and (perhaps) non-linear boundaries, linear and (perhaps) non-linear objective functions. The simplex method. Integer programming. Dynamic programming. The connection of game theory to linear programming.

However if it appears that Game Theory is the primary focus of the course, then we are dealing with the Game Theory course.

9.35 Game Theory.
Zero-sum games. Mixed strategies. Non-zero sum games such as Prisoner's dilemma. Equilibrium solutions. N-person games. Nash equilibrium. Shapley value. The connection to linear programming may be included. The distinction between this and the previous course is that in the previous course, variants of linear programming are featured and Game Theory is not a primary interest.

9.36 Dynamic Programming
Bellman equation; dynamic optimization; shortest paths; network flows; knapsack problems; sequence alignment.

9.37 Optimization/Convex Programming/Non-linear Programming
Non-linear optimization; constrained and unconstrained optimization; Lagrange multipliers, Kuhn-Tucker conditions; Hooke-Jeeves method; conjugate direction method (Powell's method); steepest descent methods.

9.38 Network Flows
Networks; flows; cuts; flow augmenting paths; max flow-min cut theorem; Dinic's algorithm; Hitchcock problem; minimum cost flows; shortest paths;

9.39 Discrete Optimization/Combinatorial Optimization
Greedy algorithms; shortest paths in directed and undirected graphs; Dikjstra's algorithm; Bellman's algorithm; breadth and depth first search; tree algorithms; minimum cost spanning trees; Kruskal's and Prim's algorithms; TSP (traveling salesman problem); network flows; heuristics; integer programming and linear programming.

9.40 Data Mining
Measurement and data; scoring functions; modeling for classification; pattern recognition; database choices; finding rules for patterns; ranking algorithms, decision tree and Bayesian learning.

9.41 Monte Carlo Methods
Simulation methods; Metropolis algorithm; Ising model; Markov chains; Gibbs sampling.

9.42 Decision Theory.
Decision trees, utilities, loss functions, risk functions, admissible decision rules, a priori distributions, Bayes decision rules, and minimax decision rules. Hypothesis testing. Game theory from t he decision-theoretic point of view. The theories of Ramsey and Savage, Bayesian inference.

Cluster: Miscellaneous applications

9.43 Mathematical Modeling.
Building and testing models in successive stages. Consideration of the fit between a model and what is being modeled. The areas of application may be diverse.

9.44 Adjustment of Observations /Least Squares.

9.45 Topics/Seminar in Applied Mathematics

9.46 Other.

Category 10: Discrete Mathematics (Worksheet Abbreviation: Disc)

Cluster: Discrete mathematics

10.1 Finite Differences/Difference Equations/Interpolation.
Difference operators. Recurrence equations of various types (linear, non-linear, homogeneous, first order, second order, etc.)

10.2 Combinatorics.
Enumeration. Systems of distinct representatives. Matroids. Stirling numbers, binomial coefficients, Bernouili numbers. Generating functions. Polynomials related to graphs.

10.3 Graph Theory.
Connectivity, trees, traversibility, coloring. Valence sequences, planarity and the genus of a graph. Digraphs.

10.4 Combinatorics and Graph Theory.
Topics from the previous two courses.

10.5 Graph Algorithms
Design of efficient algorithms for : shortest path problems, spanning tree problems, search, graph decomposition, network flow, planarity testing, etc.

10.6 Discrete Mathematics/Discrete Mathematical Structures.
Number systems and modular arithmetic, logic (propositional calculus and predicate calculus), sets and relations (equivalence relations, total and partial order relations), induction and recursion (difference equations), proof techniques (parity, mathematical induction, etc.), functions, counting (permutations, combinations, binomial coefficients), graphs with an emphasis on trees (depth- and breadth-first search), algorithms. Advanced counting (putting distinguishable and indistinguishable balls into distinguishable and indistinguishable boxes, Burnside's lemma, Polya's Theorem),

generating functions, Stirling and Catalan numbers, automata, formal languages, Boolean algebra, Boolean functions, and logic circuits, graph theory (various topics).

<u>10.7 Topics (or Seminar) in Discrete Mathematics/Combinatorics/Graph Theory</u>

<u>10.8 Other.</u>

Category 11: Advanced Probability and Statistics With Mathematics Designations[8]

(Worksheet Abbreviation: AdvProbStat)

Clustering is tricky in statistics because many courses can be taught either at a theoretical - which is to say mathematical - level, normally requiring calculus and often linear algebra, or they can be taught at an applied level where computers implement the formulas and algorithms and the emphasis is on understanding assumptions and interpreting results. As computers became more powerful and available in the second half of the 20th century, many undergraduate courses migrated from the theory cluster to the applied cluster without changing their names. Sometimes the prerequisites of the course are the best indication of which cluster it really belongs in. Our clustering is probably most appropriate for the era up to about 1975.

Cluster: Applied Statistics

<u>11.1 Design of Experiments</u>
Fundamentals of collecting data, including components of experiments, randomization and blocking, randomized design and ANOVA, multiple comparisons, power and sample size, and balanced incomplete and other block designs.

<u>11.2 Sampling Theory</u>
Sampling from finite populations; sources of sampling and estimation bias; methods of generating efficient and precise estimates of population characteristics; acceptance sampling; random and other types of sampling.

<u>11.3 Statistical Quality Control</u>

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8 Thus a course in a department of "Mathematics" or "Mathematical Science" or "Mathematics and Statistics", etc., with an "M" or "MTH" designation, as in "MTH241 Regression Analysis" is counted, whereas if, in the same department, it is "STAT241 Regression Analysis", we ignore it. If the institution does not use alphabetical prefixes or any other form of designation as to subject (using merely numbers), we count all courses listed in the department. We acknowledge that the very same course can appear with opposite designations in different times and places. Our decision to reject courses with statistics designations, no matter how mathematical their content may be, is a rough-and-ready one largely adopted to make our project manageable. However, the data we provide in this way does give a clue, albeit a highly limited one, to the relationship between mathematics and statistics, and to the views and practices of some mathematics departments in the era in question.

Fundamental concepts of quality, dimensions of quality, quality metrics, total quality management, quality improvement tools, life testing and reliability, six sigma concepts, statistical analysis of process capability, process design and improvement. concepts of process capability, modeling process capability, probability distributions (hypergeometric, binomial, Poisson, normal, Weibull), estimation of process parameters, tests of hypotheses, Pareto charts, process flow chart, cause and effect diagram (Fishbone), Taguchi methods, statistical process control (SPC), control charts, acceptance sampling, advantages and disadvantages of sampling.

<u>11.4 Data Analysis/Descriptive Statistics</u>
Methods for interpreting and understanding data; stem and leaf plots, box plots, the use of information derived from random sampling, and techniques of summarizing applications; computer intensive methods, work with actual data sets.

Cluster: Mathematical probabilitiy and statistics

<u>11.5 Theory of Probability .</u>
Sample spaces, mean, variance, conditional probabilities. Probability densities, distributions, marginal distributions. Moment generating functions, normal curves and the central limit theorem. A calculus-based course.

<u>11.6 Theory of Statistics.</u>
Hypothesis tests, confidence intervals. Regression and analysis of variance. A calculus-based course.

<u>11.7 Probability and Statistics</u>.
Combines much of the elementary material in “Theory of Probability” with that in “Theory of Statistics”.

<u>11.8 Multivariate Statistics.</u>
Using matrices and multivariable calculus, especially in connection with the multivariate normal distribution. Discriminant analysis, principal components, factor analysis, canonical correlation, multidimensional scaling.

<u>11.9 Linear Statistical Models</u>
The general linear model in matrix terms. Multiple, polynomial and stepwise regression, multicollinearity, reparametrization, normal correlation models and analysis; basic and multi-factor analysis of variance, fixed and random effects.

<u>11.10 Stochastic Processes</u>
Martingales in discrete time, stopping times, Markov chains in discrete time, continuous time stochastic processes. Diffusion. The Poisson process. Brownian motion. Stochastic integral and stochastic differential equations, their application in option-pricing.

11.11 Queueing Theory.
The modeling and analysis of queueing systems, with applications in communications, manufacturing, computer operating systems, call centers, service industries and transportation. Topics include birth-death processes and simple Markovian queues, networks of queues and product form networks, single and multi-server queues, multi-class queueing networks, fluid models, adversarial queueing networks, heavy-traffic theory and diffusion approximations.

11.12 Bayesian Statistics
Bayes theorem, subjective models for probability, Stein paradox.

11.13 Nonparametric Statistics
Goodness of fit tests (chi-squared and Kolmogorov-Smirnov tests), order and rank statistics; tests based on runs, signs, ranks, and order statistics; the two-sample; confidence and tolerance intervals, nonparametric curve estimation, Wilcoxon signed-rank tests, Mann-Whitney and Friedman tests.

11.14 Regression Analysis/Analysis of Variance
Linear models for extrapolation of data, multiple regression, logistic regression, correlation and causation. _Single and multifactor models analysis, analysis of factor effects, implementation of models, analysis of variance and of covariance.

11.15 Time Series
Forecasting; time series regression; decomposition methods; smoothing and running averages; Box-Jenkins ideas; applications to economic data and other data. May be directed to survival-model estimation if it is intended to prepare students for post-2000 actuarial exam #4. (About 30% of that exam is in time series and survival-model issues.)

11.16 Sequential Analysis
Theory of statistics when the sample size is random. Curtailed binomial sampling; Wald's sequential probability ratio test; operating characteristics, sample size and optimal properties; sequential estimation of regression function; Stein's double sampling plan; bounded length confidence intervals, selection procedures; sequential design of experiments.

Cluster: Miscellaneous probability and statistics

11.17 Topics/Seminar in Probability and Statistics.

11.18 Other.

Category 12: Computer Science Courses with Mathematics Designations Or No Designation[9]

Cluster: Programming courses

12.1 Introduction to Computing /Computer Programming
Principles of computer science; programming in FORTRAN, PL/I, C, Pascal, C++, Java or some other high level language. Algorithmic thinking and simple data structures may also be given special attention in this course.

[Note to coders: This is normally a year sequence, which we will, according to our practice, code as two instances of the same course. The standard version of this course does not especially emphasize algorithmic thinking or data structures. If these are strong features of the course, the flavor abbreviations "al" or "ds" should be used.]

12.2 Software Engineering
Software development and design principles, basics of project management, software cost estimation, object-oriented and real-time software, reliability of software, language choice.

12.3 Data Structures
Stacks, queues, linked lists, trees, heaps, sorting, recursion, trees, heaps, priority queues and hashing.

Cluster: Systems

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9 Thus a course in a department of "Mathematics" or "Mathematical Science" or "Mathematics and Computer Science", etc., with an "M" or "MTH" designation, as in "MTH241 Data Structures" is counted, whereas if, in the same department, it is "CS241 Data Structures", we ignore it. If the institution doe not use alphabetical prefixes or any other form of designation as to subject (using merely numbers), we count all courses listed in the department. We acknowledge that the very same course can appear with opposite designations in different times and schools. In the early days of computer science, the decade or so surrounding 1980, mathematics departments often took on the responsibility for this subject – often a temporary arrangement but sometimes more durable in smaller or medium-sized schools. In those early days, some mathematics departments may even have regarded computer science as one of the mathematical sciences – a view ultimately rejected by the National Science Foundation. Our decision to reject course with computer science designations, no matter how mathematical their content may be, is a rough-and-ready one largely adopted to make our project manageable. However, the data we provide in this way does give a clue, albeit a highly limited one, to the unsettled early days of computer science and to the views and practices of some mathematics departments in that era.

<u>12.4 Assembler Language Programing/Machine Organization</u>
Computer instructions and data organization, addressing concepts, data definition, binary and decimal instructions (hexadecimal), register manipulation, and linkage conventions, I/O to screen, printer, and disk interfaces.

<u>12.5 Data Communications</u>
Modems, codes, data compression, internet routing protocols

<u>12.6 Operating systems</u>
Processes, process management, synchronization, input/output devices and their programming, interrupts, memory management, resource allocation, and an introduction to file systems.

<u>12.7 Computer Architecture</u>
Boolean logic, data representation, CPU instruction sets processor organization and functional units; input/output, memory organization and hierarchy, virtual memory; system support software, and communication; data types, control unit design; buses and bus timing.

Cluster: Theory

<u>12.8 Fundamentals of Algorithms/Analysis of Algorithms</u>
Sorting, merging, searching, greedy algorithms, graph algorithms, breadth and depth first search, tree traversal. Complexity of algorithms: Polynomial time and exponential time algorithms, complexity classes, P, NP, NP-completeness, NP-hardness.

<u>12.9 Computational Complexity/Theory of Computation</u>
Models of computation, automata, languages, Turing machines, Church-Turing thesis, p polynomial time and exponential time algorithms, complexity classes, P, NP, NP-completeness, NP-hard, time-space tradeoffs, decidability.

[Note to coders: this course differs from the previous one in that it does not provide the students with their first introduction to standard commonly used algorithms such as searching and sorting.]

<u>12. 10 Automata/Computability/Formal Languages</u>
The kinds of machines (automata) that recognize various formal languages.

Cluster: Applications

<u>12.11 Computer Graphics</u>
Hidden line and hidden surface algorithms. Scan conversion, viewing models, illumination models (including some color theory), vector versus raster graphics.

12.12 Computer Vision/Image Processing
Camera models, projective geometry. Template matching, edge detection, segmentation, object recognition. Reconstructions using depth from stereo, structure from motion, and shape from shading. Properties of the human visual system, color representations, sampling and quantization, point operations, linear image filtering and correlation, transforms and subband decompositions, and nonlinear filtering, contrast and color enhancement, dithering, and image restoration, image registration.

12.13 Principles of Programming Languages
History and kinds of programming languages, object oriented, logic, and functional programming languages. Could include substantial introduction to a particular programming language.

12.14 Artificial Intelligence
Turing test, Lisp programming language, theorem proving, machine learning, game playing, self-organizing systems, rule-based systems, heuristics, expert systems.

12.15 Database Management Systems
Relational databases and index structures, entity-relationship model, functional dependency; advanced query languages, query processing and optimization, transaction processing, concurrency control, distributed databases, and database recovery, security, client server and transaction processing systems.

12.16 Compiler Design
Parsing, lexical analysis, translation specification, code generation.

12.17 Topics/Seminar in Computer Science

12.18 Other

Category 13: Courses With Unspecified Content (Worksheet Abbreviation: Unspec)

Cluster: Unspecified

13.1 Reading/Independent Study

13.2 Topics in Mathematics

13.3 Seminar course.
Like a reading course except students are expected to present material based on their reading.

13.4 Internship that carries credit.
The work is done outside the university (a company, government agency, non-profit organization, etc.)

13.5 Capstone course.
A course which serves to integrate, in the students understanding, much of what he has learned in separate conventional courses. The subject matter could range freely over various mathematical areas and will likely be different each time the course is offered.

13.6 Problem Solving.
This course has no particular subject matter, but is designed to give students experience with strategies for solving a variety of problems. Books such as Polya's "How to Solve It" or theories of problem solving may be used. In contrast to the course "Introduction to Proofs/Transition to Higher Mathematics" there is not so much emphasis on proof as on the pursuit of the "Aha!" moment.

13.7. Other

Category 14: Other Courses Not in Previous Categories (Worksheet Abbreviation: Other)

Cluster: History/Philosophy

14.1 History of Mathematics

14.2 Philosophy of Mathematics

14.3 History and Philosophy of Mathematics

Cluster: Astronomy

14.4 Descriptive Astronomy/Practical Astronomy
This course would consist of an overview of astronomy from a verbal and possibly graphic point of view and possibly including some introduction to astronomical equipment such as telescopes and sextants. It might be taught from a cultural, historic perspective and would include topics such as the solar system, planets, the Milky Way, galaxies, constellations, black holes (once these had been discovered), the sun, galaxies, stars and stellar evolution, how astronomers do science, the motions of celestial bodies,

and the fate of the universe. This course would focus on the nature of the universe and might state that it is for non-science majors.

[Note to Coder: We know from Cajori that in the late 19th century, Astronomy was taught in Mathematics Departments. This was true at the U. of Nebraska and the U. of Texas at Austin till well into the 20th century. It appears (from reading prefaces of Astronomy texts) that prior to WW II a majority of Astronomy courses were similar to this course.]

<u>14.5 Basic Mathematical Astronomy</u>
This is a mathematically based course using elementary mathematics such as arithmetic and algebra and trigonometry. Calculus would not be a prerequisite. Topics might include the universe, earth in the sky, Newton's laws of gravitation and motion, motions of the earth and moon (phases and eclipses); dimensions of the moon; motions of the planets, comets, meteors; the sun, solar structure, solar atmosphere, solar activity; coordinates on the celestial sphere, time (solar days, sidereal days, time zones, seasons), perihelion, aphelion, 23.5 degree inclination of earth's orbit, calculating the mass of earth, density of the earth, refraction and other atmospheric effects, the Foucault Pendulum (computations of differences in velocity), deflection of projectiles, precession and nutation, sidereal and synodic periods, eclipses, tides on earth, features of the moon, escape of an atmosphere, Kepler's Laws, motion of the planets, physical properties of planets, escape velocity, calculating the velocity of a comet, electromagnetic radiation, Doppler Effect, and Relativity. There would be many computational problems included in the course.

<u>14.6 Other Astronomy</u>

Cluster: Miscellaneous Other

<u>14.7 Topics/Seminar</u>

<u>14.8 Other (not astronomy)</u>