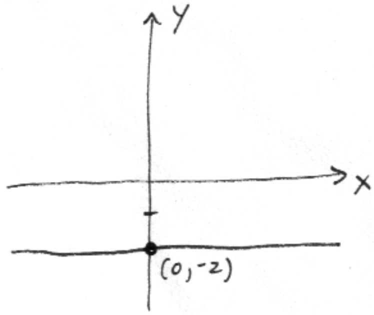
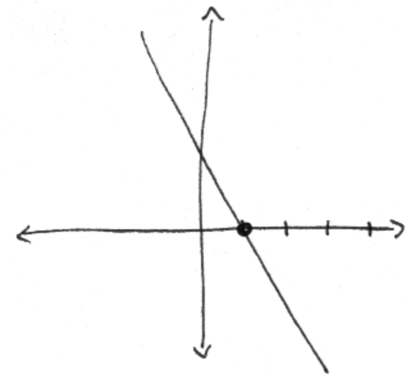
2.3 (Homework 4)

3.



4.



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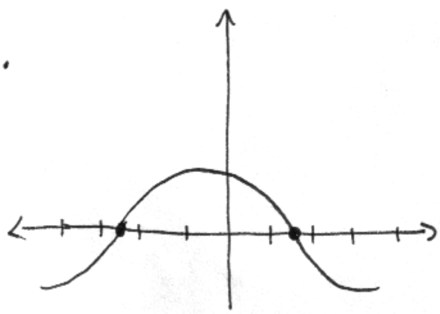

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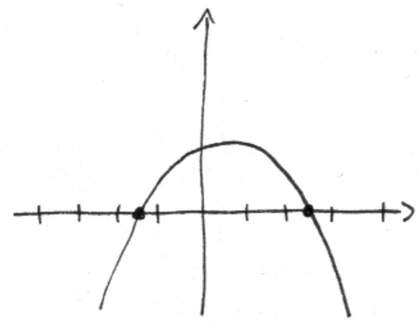
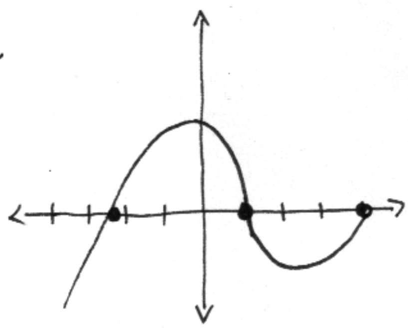
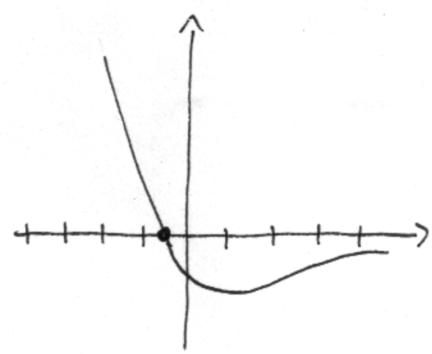


7.



8.



Use difference quotients to compute derivatives

15. $g(x) = 2x^2 - 3$

$$g'(x) = \lim_{t \to x} \frac{(2t^2-3)-(2x^2-3)}{t-x}$$

$$= \lim_{t \to x} \frac{2t^2 - 2x^2}{t-x}$$

$$= \lim_{t \to x} \frac{2(t-x)(t+x)}{t-x}$$

$$= \lim_{t \to x} 2(t+x)$$

$$= 4x$$

16. $m(x) = \frac{1}{x+1}$

$$m'(x) = \lim_{t \to x} \frac{\frac{1}{t+1} - \frac{1}{x+1}}{t-x}$$

$$= \lim_{t \to x} \left(\frac{\frac{x-t}{(t+1)(x+1)}}{t-x} \right)$$

$$= \lim_{t \to x} \frac{-1}{(t+1)(x+1)}$$

$$= \frac{-1}{(x+1)^2}$$

MAT122 Homework 4: Solutions

2.4.15.

$P = f(t) = 1.291(1.006)^t$

$f(6) = 1.291(1.006)^6 \approx 1.3382$ billion people.

$$f'(6) = \lim_{t\to 6} \frac{\Delta P}{\Delta t} = \lim_{t\to 6} \frac{1.291(1.006)^t - 1.291(1.006)^6}{t-6}$$

$$t = 5.999 : \frac{\Delta P}{\Delta t} = \frac{1.291(1.006)^{5.999} - 1.291(1.006)^6}{5.999-6} \approx 0.0080$$

$$t = 6.001 : \frac{\Delta P}{\Delta t} = \frac{1.291(1.006)^{6.001} - 1.291(1.006)^6}{6.001-6} \approx 0.0080$$

$\implies f'(6) \approx 0.0080$ billion people / year

This model estimates the population of China in 2009 to be 1.3382 billion people and increasing at a rate of 8 million people / year.

2.4.20.

$p(h) =$ pressure in dynes/cm^2 at a depth of h meters.

(a) $p(100)$ is the pressure in dynes/cm^2 at a depth of 100 meters.
(b) h such that $p(h) = 1.2 \cdot 10^6$ is the depth (in meters) where the pressure is $1.2 \cdot 10^6$ dynes/cm^2.
(c) $p(h) + 20$ is 20 dynes/cm^2 more than the pressure at h meters.
(d) $p(h + 20)$ is the pressure (in dynes/cm^2) at a depth 20 meters deeper than h.
(e) $p'(100)$ is the rate (in dynes/cm^2/m) at which the pressure increases with respect to depth when the depth is 100m.
(f) h such that $p'(h) = 20$ is the depth (in meters) such that the rate of pressure increase with respect to depth is 20 dynes/cm^2/m.

2.4.21.

$g(v)$ is fuel efficiency (mi/gal) at a speed of v mi/hr.

$g'(90)$ is measured in the units (mi/gal)/(mi/hr) = hr/gal.
$g'(55) = -0.54$ means that, when the speed is 55 mi/hr, a one unit increase in speed will result in approximately a 0.54 unit decrease in fuel efficiency.