

MAT122 - Homework 6
Due March 17

1. Consider the function f given by $f(x) = \frac{1}{x^2 + 1}$. Follow the instructions to find where f is increasing, decreasing, concave up and concave down. Then graph the function.

(a) Begin by using the quotient (or product) rule to show that

$$f'(x) = -\frac{2x}{(x^2 + 1)^2} \text{ and } f''(x) = \frac{8x^2 - 2}{(x^2 + 1)^3}$$

You'll probably have to use the chain rule as part of the process. Show your work.

(b) Next solve $f'(x) = 0$ to find where f' changes sign. Use this to determine where f is increasing or decreasing.

(c) Next solve $f''(x) = 0$ to find where f'' changes sign. Use this to determine where f is concave up or concave down.

- (d)** Find the y -intercept, x -intercepts, horizontal and vertical asymptotes. Use this information and what you learned about f in parts (a), (b) and (c) to graph $y = f(x)$.

2. Differentiate

(a) $y = 2x^6 - x + 3$

(b) $f(x) = 5^x + e^x$

(c) $2\sqrt{x} + \sqrt[3]{x}$

(d) $\ln(x) + \cos(x)$

3. Use the product rule to differentiate

(a) $\cos(x)e^x$

(b) $x^2 \ln(x)$

4. Use the quotient rule to differentiate

(a) $\frac{x+1}{x-1}$

(b) $\frac{x}{\ln(x)}$

5. Use the chain rule to differentiate

(a) $\sqrt{e^x + x}$

(b) $e^{\sqrt{x}}$

(c) $[\sin(2x + 1)]^3$