MAT122 - Homework 7 SOLUTIONS

For problems 1 through 5 use repeated applications of the chain rule to find

1. y' where $y = \sin(\sin(\sin(x)))$

$$y' = \cos(\sin(\sin(x))) \cdot \cos(\sin(x)) \cdot \cos(x)$$

2. $f'(x)$ where $f(x) = \arctan(\sqrt{x})$.

$$f'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2(1+x)\sqrt{x}}$$

3. $f'(4)$ where $f(x) = \arctan(\sqrt{x})$.

$$f'(4) = \frac{1}{2(1+4)\sqrt{4}} = \frac{1}{20}$$

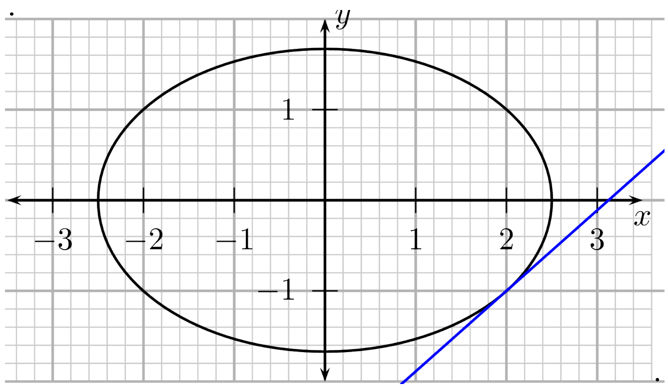
4. $\frac{da}{db}$ where $a = \ln(b^2)$.

$$\frac{da}{db} = \frac{1}{b^2} \cdot 2b = \frac{2}{b}$$

5. $\frac{d}{dx} \arcsin(\cos(y))$ assuming y is a function of x. Use trig identities to simplify the derivative.

$$\begin{aligned}
\frac{d}{dx} \arcsin(\cos(y)) &= \frac{1}{\sqrt{1-(\cos(y))^2}} \cdot (-\sin(y)) \cdot \frac{dy}{dx} \\
&= \frac{1}{\sqrt{(\sin(y))^2}} \cdot (-\sin(y)) \cdot \frac{dy}{dx} \\
&= \frac{-\sin(y)}{|\sin(y)|} \cdot \frac{dy}{dx} \\
&= \begin{cases} -\frac{dy}{dx} & \text{when } \sin(y) \geq 0 \\ \frac{dy}{dx} & \text{when } \sin(y) < 0 \end{cases}
\end{aligned}$$

For problems 6 through 9 Consider the ellipse given by $4x^2 + 9y^2 = 25$.



6. Sketch the tangent line through the point (2,-1).
7. Find the slope of the tangent line by solving $4x^2 + 9y^2 = 25$ for y and computing $y' = \frac{dy}{dx}$.

$y = \pm\frac{1}{3}\sqrt{(25 - 4x^2)}$

We use the negative solution because that describes the curve containing the point (2,-1) .

$y' = -\frac{1}{3} \cdot \frac{1}{2}\left(25 - 4x^2\right)^{-1/2}(-8x) = \frac{4}{3}x\left(25 - 4x^2\right)^{-1/2}$

$y'(2) = \frac{4}{3}(2)\left(25 - 4(2)^2\right)^{-1/2} = \frac{8}{9}$

8. Find the slope of the tangent line by differentiating $4x^2 + 9y^2 = 25$ implicitly and solving for y'.

$$\begin{aligned} 8x + 18yy' &= 0 \\ 18yy' &= -8x \\ y' &= \frac{-8x}{18y} \\ y' &= \frac{-4x}{9y} \end{aligned}$$

When $x = 2$ and $y = -1$ we have $y' = \frac{-4(2)}{9(-1)} = \frac{8}{9}$.

9. Write the equation of the tangent line to $4x^2 + 9y^2 = 25$ through the point (2,-1)

$$\begin{aligned} y &= m(x - x_1) + y_1 \\ &= \frac{8}{9}(x - 2) - 1 \end{aligned}$$

10. Use a tangent line approximation at 8 to estimate $\sqrt{70}$.

$$f(x) = \sqrt{x} \implies f'(x) = \frac{1}{2\sqrt{x}}$$

64 is close to 70, so let's make our approximation at $x = 64$.

$f'(64) = \frac{1}{2\sqrt{64}} = \frac{1}{16}$

Using the point slope equation of a line,

$$y = m(x - x_1) + y_1,$$

we find the equation of the tangent line through (64,8) to be

$$y = \frac{1}{16}(x - 64) + 8$$

We use the tangent line to make the approximation:

$$\sqrt{70} \cong \frac{1}{16}(70 - 64) + 8 = \frac{3}{8} + 8 = 8.125$$