MAT122 - Homework 8   Solutions

3.10.10.
The mean value theorem says there exists c such that

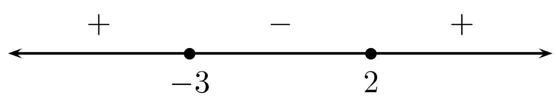
$$f'(c) = \frac{9-5}{7-2} = \frac{4}{5}$$

The book says $c = 4$. The equation of the tangent line at 4 is therefore

$$y = \frac{4}{5}(x-4) + 8$$

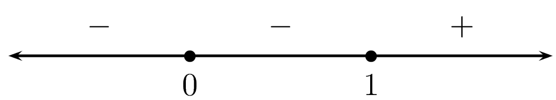
where we have used the fact that the y-coordinate at 4 is 8.

4.1.1.
$f(x) = 2x^3 + 3x^2 - 36x$
$f'(x) = 6x^2 + 6x - 36 = 0 \implies x^2 + x - 6 = 0 \implies (x+3)(x-2) = 0 \implies x = -3, 2$
The sign of f':



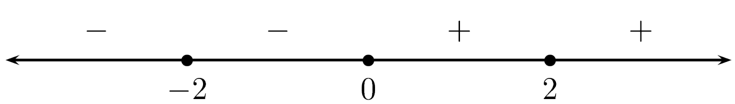
There is a local maximum at -3 and a local minimum at 2.

4.1.2.
$f(x) = 3x^4 - 4x^3 + 6$
$f'(x) = 12x^3 - 12x^2 + 6 = 0 \implies 2x^3 - 2x^2 = 0 \implies 2x^2(x-1) = 0 \implies x = 0, 1$
The sign of f':



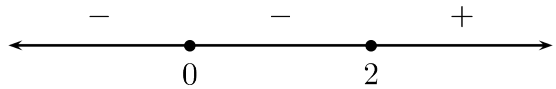
There is a local minimum at 1.

4.1.3.
$f(x) = (x^2 - 4)^7$
$f'(x) = 7(x^2 - 4)^6 \cdot 2x = 0 \implies x = 0, \pm 2$
The sign of f':



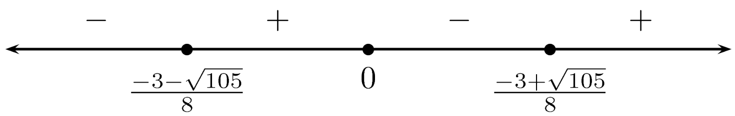
There is a local minimum at 0.

4.1.4.
$f(x) = (x^3 - 8)^4$
$f'(x) = 4(x^3 - 8)^3 \cdot 3x^2 = 0 \implies x = 0, 2$
The sign of f':



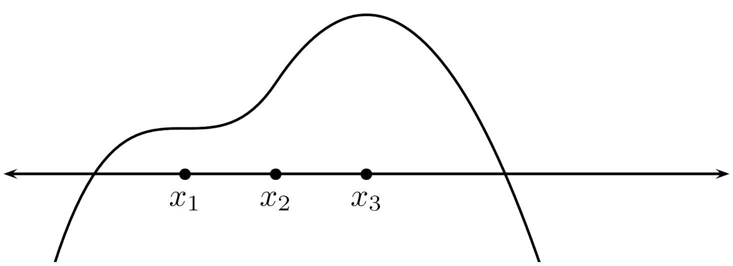
There is a local minimum at 2.

4.1.13.
$f(x) = x^4 + x^3 - 3x^2 + 2$
$f'(x) = 4x^3 + 3x^2 - 6x = 0 \implies x(4x^2 + 3x - 6) = 0 \implies x = 0$ or $x = \frac{-3 \pm \sqrt{105}}{8}$
The sign of f':

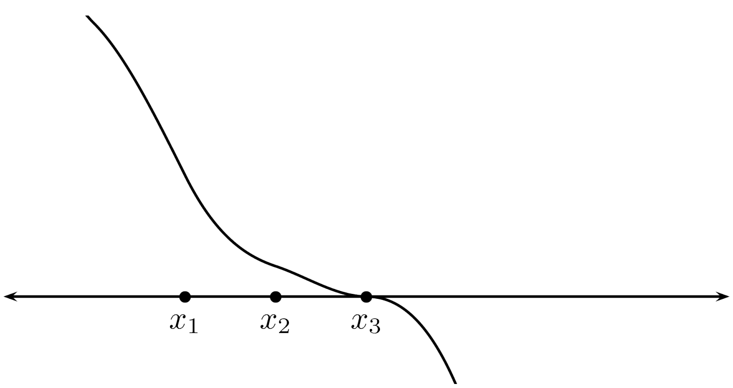


There are local minima at $x = \frac{-3-\sqrt{105}}{8}$ and $x = \frac{-3+\sqrt{105}}{8}$.
There is a local maximum at $x = 0$.

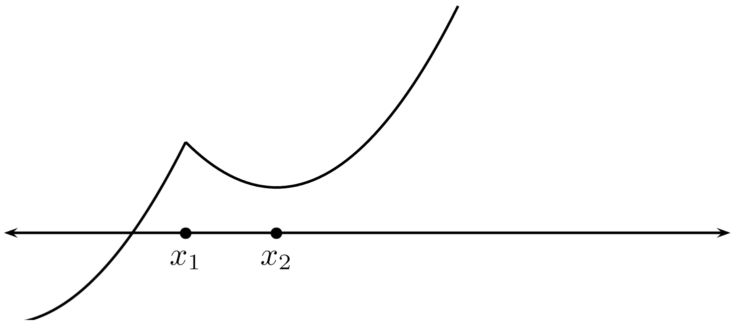
4.1.28.



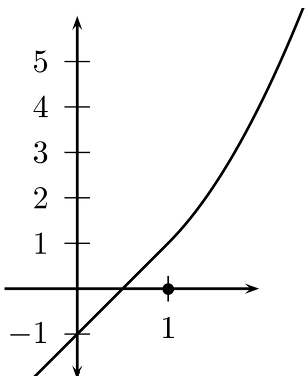
4.1.29.



4.1.30.



4.1.31.



Here, $x_1 = 1$ and the function is given by

$$f(x) = \begin{cases} 2x - 1 & : x \leq 1 \\ x^2 & : x > 1 \end{cases}$$