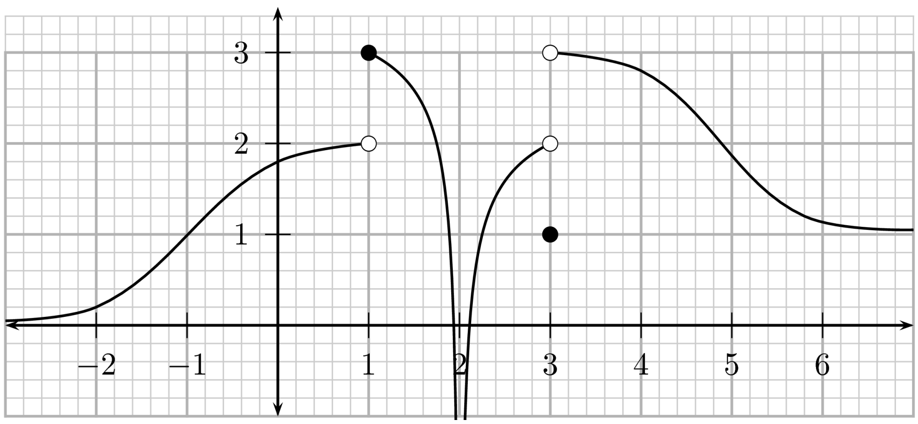
MAT122
Final Exam Practice Problems
(last updated May 6)

1. Compute the following given the graph of f below. Write "DNE" if it does not exist.

(a) $\lim_{x\to\infty} f(x)$ **(b)** $\lim_{x\to-\infty} f(x)$ **(c)** $\lim_{x\to 2} f(x)$

(d) $\lim_{x\to 1^-} f(x)$ **(e)** $\lim_{x\to 3^+} f(x)$ **(f)** $\lim_{x\to 3} f(x)$

(g) $\lim_{x\to -1} f(x)$ **(h)** $f(3)$ **(i)** $f(1)$



2. Use L'Hopital's rule to compute the following:

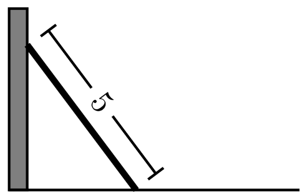
(a) $\lim_{x\to\infty} \frac{3x^2+x}{5-6x^2}$ **(b)** $\lim_{x\to 1} \frac{\ln(x)}{x-1}$ **(c)** $\lim_{x\to\infty} x^2e^{-x}$

3. Suppose $f(x) = \ln(x)$.
(a) Express $f'(1)$ as a limit using the definition of derivative.
(b) Evaluate the limit.

4. Suppose $f(x) = \sqrt{1+x^2}$.
(a) Compute $f'(x)$.
(b) Compute $f''(x)$.
(c) Give the intervals on which f is increasing, decreasing, concave up, and concave down.

5. Consider the curve given by $x^2 - xy + y^2 = 7$.
(a) Use implicit differentiation to find the slope of the tangent line through the point (1,3).
(b) Write the equation of the tangent line through (1,3)
(c) Use the equation of the tangent line to approximate the y-coordinate of the point on the curve with x-coordinate 1.2.

6. Consider a 5 meter ladder leaning against a wall. Suppose the top of the ladder slides down the wall at a constant rate of -1m/s (negative because the height is decreasing). At what rate is the bottom of the ladder moving away from the wall at the instant it is 3m from the wall?



7. The velocity (m/s) of an object at various times *t*, in seconds (s), is given by the following table:

t	0	1	2	3	4
v	16	9	4	1	0

(a) Estimate the total displacement of the object for the time interval [0,4] using a left-hand sum.
(b) Estimate the total displacement of the object for the time interval [0,4] using a right-hand sum.
(c) Suppose that the velocity is given by the function $v(t) = 16 - t^2$. Compute the exact displacement of the object for the time interval [0,4].

8. Use substitution to evaluate the following

(a) $\int_0^{\pi/2} \cos(x)e^{\sin(x)}\,dx$ **(b)** $\int_{-2}^{2} \frac{x}{\sqrt{1+x^2}}\,dx$ **(c)** $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

9. A rectangle is drawn from the origin (0,0) to a point (*x*,*y*) on the parabola given by $y = 2x - \frac{1}{2}x^2$. Find the *positive* coordinates *x* and *y* which maximize the rectangle's area.