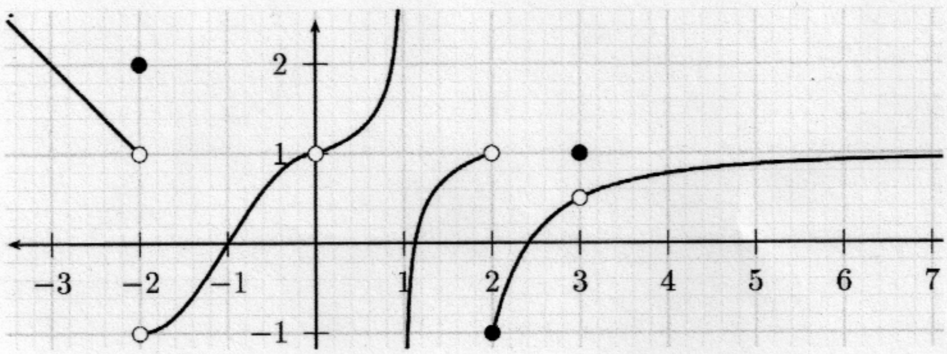
MAT122
Exam 1

Name KEY Student ID Number:

Instructions

- *There are a total of 8 questions totaling 60 points possible.*
- *Be sure to show your reasoning for any answers which are not obvious or which require multiple steps. A correct answer without justification may not earn you any points. Moreover, incorrect answers preceded by valid reasoning may earn you partial credit.*
- *Please silence your cell phone.*
- *Raise your hand if you have any questions.*
- *Bring your exam to the front when completed.*

1. *(12pts)* Let $f(x)$ be the function whose graph is shown below.



Compute the following limits. If it does not exist write "DNE".

(a) $\lim_{x\to 3} f(x)$ 1.5

(b) $\lim_{x\to 1^-} f(x)$ ∞

(c) $\lim_{x\to -2^-} f(x)$ 1

(d) $\lim_{x\to 0} f(x)$ 1

(e) $\lim_{x\to 2} f(x)$ DNE

(f) $\lim_{x\to \infty} f(x)$ 1

2. *(2pts)* List all values for which the function above is *not* continuous.

−2, 0, 1, 2, 3

3. *(12pts)* Compute the following limits:

(a) $\lim\limits_{x\to-\infty} \dfrac{e^x}{x}$ $\quad e^x \to 0,\ x \to -\infty$ $\quad = \boxed{0}$

(b) $\lim\limits_{x\to5^-} \dfrac{1}{5-x}$ $\quad = \boxed{+\infty}$

$+\infty$ since $5-x > 0$ when $x<5$

(c) $\lim\limits_{x\to2} \dfrac{x-2}{x^2-4}$ $= \lim\limits_{x\to2} \dfrac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$

$= \lim\limits_{x\to2} \dfrac{1}{x+2} = \boxed{\dfrac{1}{4}}$

(d) $\lim\limits_{x\to\infty} \dfrac{5-12x^5}{3x^5+8x^2}$ $= \lim\limits_{x\to\infty} \dfrac{5/x^5 - 12}{3 + \frac{8x^2}{x^5}}$

$= \dfrac{0-12}{3+0} = \boxed{-4}$

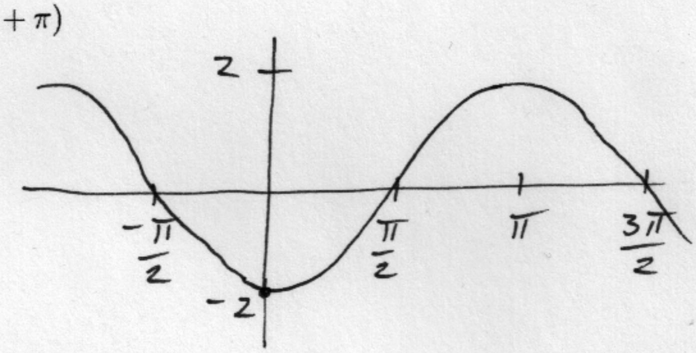
(e) $\lim\limits_{x\to0^+} \ln(x)$ $\quad = -\infty$

(f) $\lim\limits_{x\to\infty} \arctan(x)$ $\quad = \dfrac{\pi}{2}$

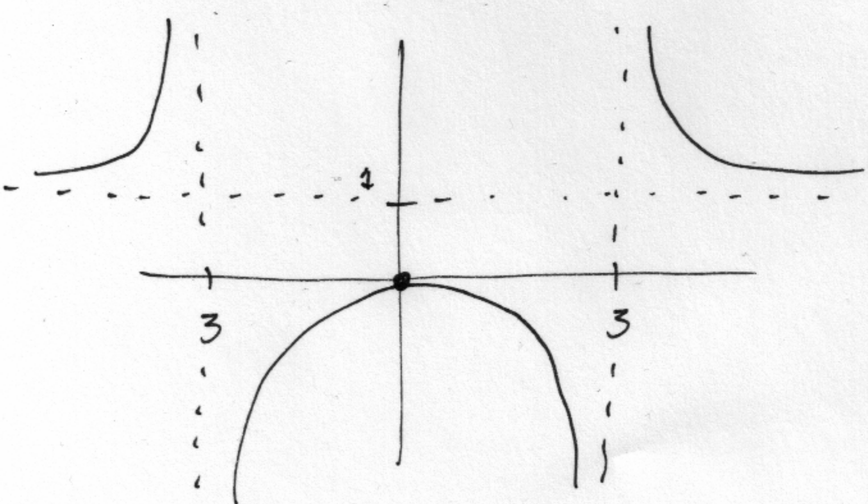
see graph

4. *(6pts)* Graph the functions. Indicate and label any *x*-intercepts and asymptotes.

(a) $2\cos(x+\pi)$



(b) $\dfrac{x^2}{x^2-9}$ $\qquad$ (Hint: Use the fact that $\dfrac{x^2}{x^2-9} > 1$ when $x > 3$ and when $x < -3$.)



5. **(a)** *(3pts)* Solve for x using log base 3, $\log_3$.

$$27^{x+1} = 9^{x-2}$$

$$\log_3 27^{x+1} = \log_3 9^{x-2}$$

$$(x+1)\log_3 27 = (x-2)\log_3 9$$

$$3(x+1) = 2(x-2)$$

$$3x+3 = 2x-4$$

$$\boxed{x = -7}$$

(b) *(3pts)* Solve for r.

$$\ln(r+1) + \ln(r-1) = \ln(8)$$

$$\ln[(r+1)(r-1)] = \ln 8$$

$$e^{\ln(r^2-1)} = e^{\ln 8}$$

$$r^2 - 1 = 8$$

$$r^2 = 9$$

$$r = \pm 3$$

reject negative solution since not in domain of $\ln(r-1)$.

$$\boxed{r=3}$$

6. *(4pts)* Use the intermediate value theorem to show that

$$p(x) = x^4 - 3x^3 + x^2 + 3$$

has a root between 1 and 2.

p is continuous since it is a polynomial.

$p(1) = 1 - 3 + 1 + 3 = 2$

$p(2) = 16 - 24 + 4 + 3 = -1$

0 is between $p(1)$ and $p(2)$.
Therefore, by the intermediate value theorem, $p(x) = 0$ has a solution between 1 and 2.

7. *(12pts)* $f(x) = x^2 + 1$. Use the limit definition of derivative to solve the following.

(a) Find $f'(5)$.

$$f'(5) = \lim_{t \to 5} \frac{(t^2+1)-(5^2+1)}{t-5} = \lim_{t \to 5} \frac{t^2-25}{t-5}$$

$$= \lim_{t \to 5} \frac{(t+5)\cancel{(t-5)}}{\cancel{t-5}}$$

$$= \lim_{t \to 5} t+5 = \boxed{10}$$

(b) Write the derivative as a function of x. That is, find $f'(x)$.

$$f'(x) = \lim_{t \to x} \frac{(t^2+1)-(x^2+1)}{t-x} = \lim_{t \to x} \frac{t^2-x^2}{t-x}$$

$$= \lim_{t \to x} \frac{(t+x)\cancel{(t-x)}}{\cancel{t-x}}$$

$$= \lim_{t \to x} t+x = \boxed{2x}$$

(c) If $f(x)$ gives the displacement in meters. at time x, measured in seconds, find the instantaneous velocity at time 13 seconds.

$$f'(13) = 2 \cdot 13 = \boxed{26 \ m/s}$$

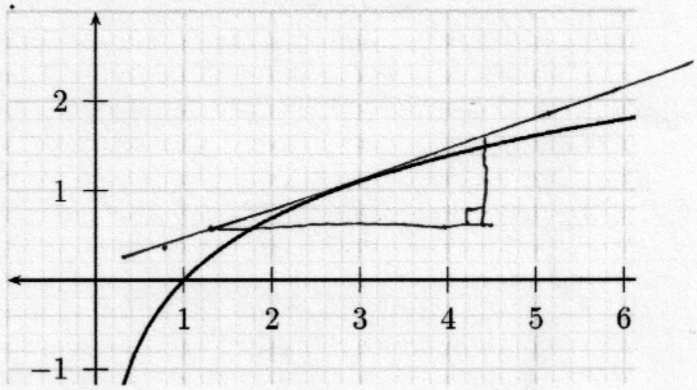
(d) Find the average velocity on the time interval [0,3].

$$V_{AVE} = \frac{f(3)-f(0)}{3-0} = \frac{(3^2+1)-(0^2+1)}{3-0}$$

$$= \frac{10-1}{3}$$

$$= \boxed{3} \ m/s$$

8. *(6pts)* Let $f(x)$ be the function whose graph is shown below.



a. Using a straight edge, draw a tangent line through the point $(3, f(3))$.

b. Use the tangent line to estimate $f'(3)$.

$$f'(3) = \frac{\text{rise}}{\text{run}} \approx \boxed{\frac{1}{3}}$$