MAT122
Practice Exam 2

1. Consider the function f given by

$$f(x) = x^{4/3} + \frac{x}{3}$$

Use a linear approximation at $x = 27$ to estimate $f(30)$.

2. Use the first derivative test to find the local minima and maxima of

$$f(x) = 3x^4 - 8x^3 + 6x^2 + 13$$

3. Use the second derivative test to find the local minima, maxima, and inflection points of

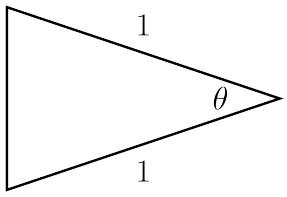
$$f(x) = x^4 - 18x^2$$

4. Consider the curve given by the equation

$$y^2(x + y - 1) - 2x^3 = 0.$$

Write the equation of the tangent line through the point (0,1).

5. Find the angle θ which maximizes the area of the isosceles triangle below.



.

Hint: First use trigonometry to show that the area can be expressed as $\frac{1}{2}\sin(\theta)$.

6. Consider a spherical tank of radius 1m which is being filled with water at a rate of 3 m^3/min. If you take take calculus II, you'll learn how to show that the volume of water at height *h* is given by $\pi(h^2 - \frac{1}{3}h^3)$. At what rate is the height of water increasing at the instant the height of water is $\frac{1}{2}$m?

7. Suppose the spherical tank above began empty and was half full 1 minute later, use the mean value theorem to argue that at some point the water was flowing in at a rate of at least 2 m^3/min.

8. Compute the derivatives of the following expressions. Simplify your answers.

(a) $\arcsin(3x^2)$

(b) $\ln(\sin(x))$

(c) $\dfrac{x-1}{x+1}$

(d) $x^3 e^{2x}$

(e) $5x^7 - 4x - \dfrac{3}{x} + 8$

(f) $\sqrt{x} - \sqrt[3]{x}$