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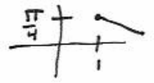
Limit definition of continuity:

- f is <u>continuous</u> at a if and only if $\lim_{x\to a} f(x) = f(a)$.

- f is continuous in an open interval provided f is continuous at each number in that interval.



<u>Example</u>: Show that $f(x) = \begin{cases} \arctan(x) & \text{if } x < 1 \\ \frac{\pi}{2} - \frac{\pi}{4}x & \text{if } x \geq 1 \end{cases}$ is continuous.

- f is continuous on $(-\infty, 1)$
- f is continuous on $(1, \infty)$
- f is continuous at 1:

$$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} \arctan x = \frac{\pi}{4} \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1} \frac{\pi}{2} - \frac{\pi}{4}x = \frac{\pi}{4} \end{aligned}\right\} \Rightarrow \lim_{x\to 1} f(x) = \frac{\pi}{4} = f(1)$$

<u>Practice</u>: Find

$$\lim_{x\to 3} \frac{x^2+2}{x-1}$$

$$\lim_{x\to 2} \frac{x^2-4}{x-2}$$

- limits of the form $\frac{0}{0}$ must be simplified.

$$\lim_{x\to 0} \frac{\sqrt{3} - \sqrt{x}}{3-x}$$

$$\lim_{x\to 0} \frac{\cos x}{\sqrt{x^2+e^x}}$$

<u>Limits involving ∞:</u>

$\lim_{x\to a} f(x) = \infty$ $(-\infty)$ means that no matter how large a positive (negative) number we choose, $f(x)$ will be at least that large provided x is suff. close to a.

$$\lim_{x\to 0} \frac{1}{x^2} = +\infty$$

$$\lim_{x\to \frac{\pi}{2}^+} \tan x = -\infty$$

$$\lim_{x\to 2^-} \frac{x+2}{x^2-4x+4} = -\infty$$