

MAT 123 - Homework 1, Solutions

4. (a) Find the slope of the line through the points (-2,5) and (3,4).

$$m = \frac{4-5}{3-(-2)} = -\frac{1}{5}$$

- (b) Write the equation of this line.

$$y - y_1 = m(x - x_1) \Rightarrow y - 4 = -\frac{1}{5}(x - 3)$$

- (c) Write the equation of a line with slope 0.

$$y = \pi$$

5. Compute

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{x - 3} = \lim_{x \rightarrow 3} x + 3 = 6$

(b) $\lim_{t \rightarrow \infty} \arctan(t) = \pi/2$

(c) $\lim_{x \rightarrow -\infty} \frac{e^x}{x^2} = 0$ since $e^x \rightarrow 0$ and $x^2 \rightarrow \infty$.

6. (a) $y = x^3 - x + 2$. Find $\frac{dy}{dx}$. $\frac{dy}{dx} = 3x^2 - 1$

(b) $f(t) = \cos(t)$. Find $f'(t)$. $f'(t) = -\sin(t)$

(c) $g(x) = \ln(3x)$. Find $\frac{d}{dx}g(x)$. $\frac{d}{dx}g(x) = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

(d) $x^2 + y^2 = 25$. Find dy/dx when $x = 3$ and $y = 4$.

$$2x \frac{dx}{dx} + 2y \frac{dy}{dx} = 0 \Rightarrow 2x + 2yy' = 0 \Rightarrow y' = -x/y$$

7. From physics we know that the displacement d of an object under the influence of a constant acceleration a is given by

$$d(t) = \frac{1}{2}at^2 + v_0t + d_0,$$

where d_0 denotes initial displacement, v_0 denotes initial velocity and t denotes time.

Suppose a ball is tossed from an initial height of 2m into the air with initial velocity 5 m/s and is affected by a constant downward acceleration of -9.8 m/s^2 due to gravity.

(a) How long does it take for the ball to reach its maximum height?

At the maximum height velocity is zero:

$$d'(t) = at + v_0 = 0 \Rightarrow -9.8t + 5 = 0 \Rightarrow t = 25/49$$

(b) What is the maximum height of the ball?

$$d\left(\frac{25}{49}\right) = \frac{1}{2}(-9.8)\left(\frac{25}{49}\right)^2 + 5\left(\frac{25}{49}\right) + 2 = -\frac{125}{98} + \frac{125}{49} + 2 = 321/98$$

8. Compute the following

(a) $\int x^2 dx = \frac{1}{3}x^3 + C$

(b) $\int_0^1 x^2 dx = \left[\frac{1}{3}x^3\right]_0^1 = 1/3$

(c) $\int_{-3}^3 \frac{y^3}{y^2 + 1} dy$ (Hint: graphing the integrand on a calculator may help.)

$$\int_{-3}^3 \frac{y^3}{y^2 + 1} dy = 0 \text{ since } \frac{y^3}{y^2 + 1} \text{ is an odd function.}$$