MAT123 - Homework 2 solutions

Use substitution to simplify

1. $\displaystyle\int_1^2 \frac{1}{2-3x}dx$. Let $u = 2-3x$. Then $du = -3dx \Rightarrow dx = -\frac{1}{3}du$.

$$\int_1^2 \frac{1}{2-3x}dx = \int_{-1}^{-4} \frac{1}{u}\left(-\frac{1}{3}\right) du = -\frac{1}{3}\left[\ln|u|\right]_{-1}^{-4} = -\frac{1}{3}\ln(4)$$

2. $\displaystyle\int \cos(x)\sin(\sin(x))dx$. Let $u = \sin(x)$. Then $du = \cos(x)dx$.

$$\int \cos(x)\sin(\sin(x))dx = \int \sin(u)du = -\cos(u) = -\cos(\sin(x)) + C$$

Use integration by parts to simplify

3. $\displaystyle\int t^{3/2}\ln(t)dt$

$u = \ln(t) \qquad dv = t^{3/2}dt$

$du = \frac{1}{t}dt \qquad v = \frac{2}{5}t^{5/2}$

$$\int t^{3/2}\ln(t)dt = \ln(t)\frac{2}{5}t^{5/2} - \int \frac{2}{5}t^{5/2}\frac{1}{t}dt = \frac{2}{5}\ln(t)t^{5/2} - \frac{2}{5}\int t^{3/2}dt$$
$$= \tfrac{2}{5}\ln(t)t^{5/2} - \tfrac{4}{25}t^{5/2} + C$$

4. $\displaystyle\int_0^5 ze^{-0.6z}dz = \int_0^5 ze^{-\frac{3}{5}z}dz$

$u = z \qquad dv = e^{-\frac{3}{5}z}dz$

$du = dz \qquad v = -\frac{5}{3}e^{-\frac{3}{5}z}$

$$\int_0^5 ze^{-\frac{3}{5}z}dz = \left[-\frac{5}{3}ze^{-\frac{3}{5}z} + \int \frac{5}{3}e^{-\frac{3}{5}z}\,dz\right]_0^5 = \left[-\frac{5}{3}ze^{-\frac{3}{5}z} - \frac{25}{9}e^{-\frac{3}{5}z}\right]_0^5 = \frac{25}{9} - \frac{100}{9}e^{-3}$$

Use the table of integrals to simplify

5. $$\int (x-2)^5\sin(2x)dx = -\frac{1}{2}(x-2)^5\cos(2x) + \frac{5}{4}(x-2)^4\sin(2x) + \frac{5}{2}(x-2)^3\cos(2x)$$
$$-\frac{15}{4}(x-2)^2\sin(2x) - \frac{15}{4}(x-2)\cos(2x) + \frac{15}{8}\sin(2x)$$

6. $\displaystyle\int \frac{1}{\sqrt{4x^2+9}}dx$ Let $u = 2x$. Then $du = 2dx$.

$$\int \frac{1}{\sqrt{4x^2+9}}dx = \frac{1}{2}\int \frac{1}{\sqrt{u^2+9}}du = \frac{1}{2}\ln|u+\sqrt{u^2+9}| + C$$
$$= \frac{1}{2}\ln|2x+\sqrt{4x^2+9}| + C$$

7. $\displaystyle\int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-2)(x-3)}dx$

$$\frac{1}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2} \Rightarrow 1 = A(x-2) + B(x-3)$$

$x^0 : 1 = -2A - 3B$
$x^1 : 0 = A + B$

$\Rightarrow B = -A$ and $1 = -2A + 3A = A \Rightarrow B = -1$

$$\Rightarrow \int \frac{1}{x^2-5x+6}dx = \int \frac{1}{(x-2)(x-3)}dx = \int \frac{1}{x-3} - \frac{1}{x-2}dx$$
$$= \ln|x-3| - \ln|x-2| + C$$
$$= \ln\left|\frac{x-3}{x-2}\right| + C$$

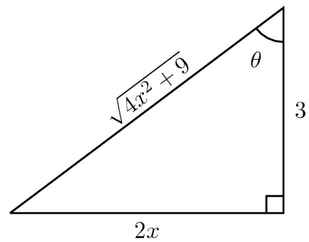
Use long division to simplify:

8. $\displaystyle\int \frac{x^4+4}{x^2+4}dx = \int x^2 - 4 + \frac{20}{x^2+4}dx = \frac{1}{3}x^3 - 4x + 20\left[\frac{1}{2}\arctan\left(\frac{x}{2}\right)\right] + C$

$$= \frac{1}{3}x^3 - 4x + 10\arctan\left(\frac{x}{2}\right) + C$$

Use trig substitution to simplify

9. $\displaystyle\int \frac{1}{4x^2+9}dx$ Let $\tan\theta = \frac{2x}{3}$. Then $\cos(\theta) = \frac{3}{\sqrt{4x^2+9}}$ and $dx = \frac{3}{2}\sec^2\theta\, d\theta$



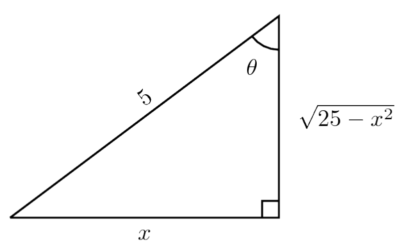
So $\displaystyle\int \frac{1}{4x^2+9}dx = \int \frac{1}{9}\cos^2\theta\frac{3}{2}\cdot\sec^2\theta\, d\theta = \int \frac{1}{6}d\theta = \frac{1}{6}\theta + C$

$$= \frac{1}{6}\arctan\left(\frac{2x}{3}\right) + C$$

10. Find the area of the ellipse given by $x^2 + 16y^2 = 25$.

One quarter of the ellipse is given by $y = \frac{1}{4}\sqrt{25-x^2}$ on the interval [0,5]. Therefore,

Area = $\displaystyle 4\int_0^5 \frac{1}{4}\sqrt{25-x^2}\, dx = \int_0^5 \sqrt{25-x^2}\, dx$



Use the trig substitution $x = 5\sin\theta \Rightarrow dx = 5\cos\theta d\theta$, and from the triangle we see that $5\cos\theta = \sqrt{25-x^2}$. Thus,

Area = $\displaystyle\int_0^5 \sqrt{25-x^2}\, dx = \int_0^{\pi/2} 25\cos^2\theta d\theta = 25\left[\frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\int d\theta\right]_0^{\pi/2} = \frac{25\pi}{4}$,

where the second to last equality is from the table of integrals (but is easily derivable using the half angle formula $\cos(2\theta) = 2\cos^2\theta - 1$).