

MAT123**Extra Credit. Due March 31.**

Solutions must be neat and stapled. Present each answer on a separate sheet. Each correct solution will add 3 points (7.5%) to your first midterm. Please come see me for hints if you get stuck.

1. A solid is generated by rotating about the x-axis the region bounded by the x-axis, the y-axis, and the curve $y = f(x)$, $x \geq 0$, where f is a positive function. The volume generated by the part of the curve from $x = 0$ to $x = b$ is b^2 for all $b > 0$. Find the function f .

2. (a) Show that for $x > 0$,

$$\frac{x}{1+x^2} < \arctan(x) < x.$$

(b) Show that for $x > 0$ and $x \neq 1$,

$$\ln(x) < x - 1.$$

(c) Show that for $x > 0$ and $x \neq 1$,

$$\frac{x-1}{x} < \ln x.$$

3. Show that

$$\cos\{\arctan[\sin(\operatorname{arccot} x)]\} = \sqrt{\frac{x^2+1}{x^2+2}}$$

4. Show which is bigger without using a calculator: e^π or π^e .