

**MAT123**  
**Exam 3 Practice Problems**  
**Updated May 5**

**Don't panic. The actual final exam will not be this long. But do study hard!!**

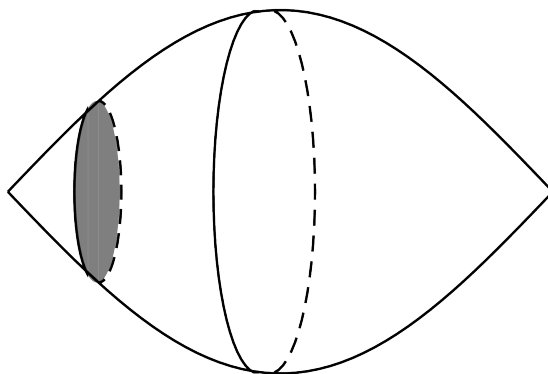
1. Use the comparison test to determine whether the following improper integrals converge or diverge.

$$\int_0^5 \frac{2}{\sqrt{x+x^2}} dx$$

$$\int_1^{\infty} \frac{e^{-x}}{\sqrt{x+x^2}} dx$$

$$\int_0^1 \frac{1}{\sqrt{9x^2-x^3}} dx$$

2. Find the volume obtained by rotating the region bounded by the curves  $y = x^2$  and  $y = \sqrt{x}$
- (a) about the  $x$ -axis.  
(b) about the  $y$ -axis.
3. Find the area bounded between the curves  $3 - x = 3y^2$  and  $x + 5 = 5y^4$ . They intersect at  $(0,1)$  and
4. A cylindrical tank with radius 2ft and height 5ft is standing on its circular base and is filled with a fluid having density of  $10 \text{ lb/ft}^3$ . Compute the force exerted by the fluid on the side of the tank.
5. Write an integral which gives the arc length of the curve given by  $y = \ln(x)$  between the points  $(1,0)$  and  $(e,1)$ . Do not attempt to evaluate the integral.
6. Find the volume of region below whose cross-sectional area (the area of the shaded region) at distance  $x$  from one end is given by  $A(x) = \sin(x)$ .



7. Find the limit of the series  $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}}$ .

8. Explain why the following series converge or diverge:

(a)  $\sum_{n=0}^{\infty} \sqrt{\frac{n+1}{n}}$

(b)  $\sum_{n=0}^{\infty} \left(\frac{-4}{3+n}\right)^n$

(c)  $\sum_{n=0}^{\infty} \frac{5^n}{n!}$

9. Suppose a function  $f$  gives rise to the following table of values:

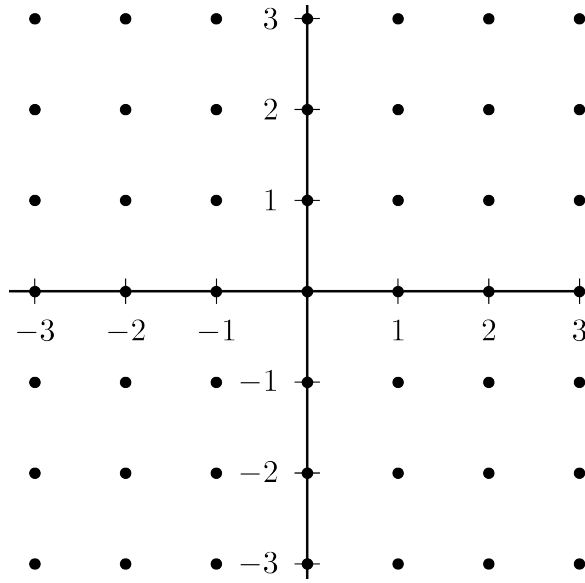
$x$	$f^{(0)}(x)$	$f^{(1)}(x)$	$f^{(2)}(x)$	$f^{(3)}(x)$
0	2	3	0	-1
2	5	0	-4	1

(a) Give the 3<sup>rd</sup> order Taylor approximation to  $f$  centered at 0.

(b) Give the 3<sup>rd</sup> order Taylor approximation to  $f$  centered at 2.

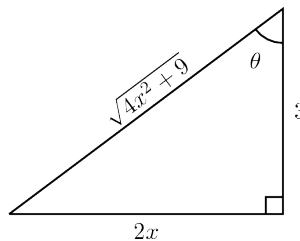
10. Use the fact that  $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$  to write the Taylor series for  $\arctan(x)$  centered at 0.

11. Graph the slope field for  $y' = \frac{x+y}{2}$  using the grid below. Draw one line segment through each grid point.



Use Euler's Method with step size 1 to estimate  $y(2)$  where  $y$  is the solution which satisfies  $y(-2) = 1$ .

12. Compute the numerical value of  $\int_3^{\infty} \frac{-2x}{(1+x^2)^2} dx$ .
13. Use the fact that  $\frac{1}{x^2 - 3x - 10} = \frac{A}{x - 5} + \frac{B}{x + 3}$  to find  $\int \frac{1}{x^2 - 3x - 10} dx$ . You must determine the values of  $A$  and  $B$ .
14. Use the substitution suggested by the triangle below to compute  $\int \frac{9}{4x^2 + 9} dx$ .



15. Use the method of separation of variables to solve the differential equation  $y' = x + xy$ .
16. Verify that  $y = Ae^x + Be^{2x}$  is a solution to  $y'' - 2y' + e^x = 0$ . Find  $A$  and  $B$  so that  $y(0) = 1$  and  $y'(0) = 3$ .
17. Use integration by parts twice to compute  $\int x^2 e^x dx$ .