

$$7. \quad \int \frac{v^2 + 2v + 1}{v(v^2 + 1)} dv$$

$$\frac{v^2 + 2v + 1}{v(v^2 + 1)} = \frac{A}{v} + \frac{Bv + C}{v^2 + 1}$$

$$\begin{aligned} v^2 + 2v + 1 &= A(v^2 + 1) + (Bv + C)v \\ &= Av^2 + A + Bv^2 + Cv \end{aligned}$$

$$v^2 : 1 = A + B$$

$$v^1 : 2 = C$$

$$v^0 : 3 = A$$

$$\Rightarrow B = 0$$

$$\Rightarrow \int \frac{v^2 + 2v + 1}{v(v^2 + 1)} dv = \int \frac{1}{v} + \frac{2}{v^2 + 1} dv = \ln |v| + 2 \arctan(v) + K$$

10. Use the trapezoid, midpoint, and Simpson's rule, each with 4 subdivisions, to estimate

$$\int_{-3}^5 2^x dx.$$

$$\begin{aligned} \text{TRAP}(4) &= \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)] \\ &= \frac{2}{2} [2^{-3} + 2 \cdot 2^{-1} + 2 \cdot 2^1 + 2 \cdot 2^3 + 2^5] \\ &= 53.125 \end{aligned}$$

$$\begin{aligned} \text{MID}(4) &= \Delta x [f(x_0^*) + f(x_1^*) + f(x_2^*) + \cdots + f(x_{n-1}^*)] \\ &= 2 [2^{-2} + 2^0 + 2^2 + 2^4] \\ &= 42.5 \end{aligned}$$

$$\begin{aligned} \text{SIMP}(4) &= \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)] \\ &= \frac{2}{3} [2^{-3} + 4 \cdot 2^{-1} + 2 \cdot 2^1 + 4 \cdot 2^3 + 2^5] \\ &= 46.75 \end{aligned}$$