MAT123
Exam 1

Name KEY Student ID Number:

Instructions

- *Be sure to show your reasoning. A correct answer without justification may not earn you any points. Moreover, incorrect answers preceded by valid reasoning may earn you partial credit.*
- *Please silence your cell phone.*
- *Raise your hand if you have any questions.*
- *Bring your exam to the front when completed.*

1. Use substitution to find $\int_0^{\pi/2} 3(\sin(\theta))^2 \cos(\theta)\, d\theta$.

$u = \sin\theta$ $\qquad \theta = 0 \Rightarrow u = 0$

$du = \cos\theta\, d\theta$ $\qquad \theta = \pi/2 \Rightarrow u = 1$

$$\int_0^1 3u^2\, du = u^3\Big|_0^1 = (1)^3 - (0)^3 = \boxed{1}$$

2. Use integration by parts to find $\int_0^2 xe^x\, dx$.

$u = x$ $\qquad dv = e^x\, dx$

$du = dx$ $\qquad v = e^x$

$$\left[xe^x - \int e^x\, dx\right]_0^2 = \left[xe^x - e^x\right]_0^2$$

$$= (2e^2 - e^2) - (0\cdot e^0 - e^0)$$

$$= \boxed{e^2 + 1}$$

3. Apply the method of integration by parts twice to find $\int \sin(y) \cdot e^y \, dy$

$u = \sin y \qquad dv = e^y \, dy$

$du = \cos y \, dy \qquad v = e^y$

$\sin y \, e^y - \int e^y \cos y \, dy$

$w = \cos y \qquad dt = e^y \, dy$

$dw = -\sin y \, dy \qquad t = e^y$

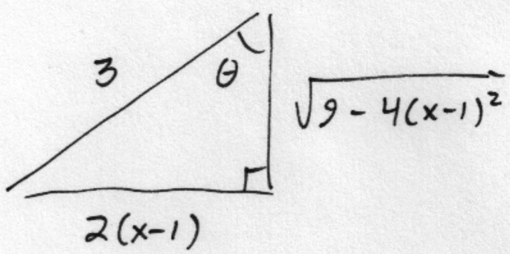
$$\int \sin(y) e^y dy = \sin y \, e^y - \left[\cos y \, e^y - \int -\sin y \cdot e^y dy \right]$$

$$= \sin y \, e^y - \cos y \cdot e^y - \int \sin(y) \, e^y \, dy$$

$$2\int \sin(y) e^y dy = e^y (\sin y - \cos y)$$

$$\boxed{\int \sin(y) e^y dy = \frac{1}{2} e^y (\sin y - \cos y) + K}$$

4. Use trigonometric substitution to find $\int \frac{2dx}{\sqrt{9 - 4(x-1)^2}}$.



$3 \sin\theta = 2(x-1)$

$3 \cos\theta = \sqrt{9 - 4(x-1)^2}$

$3 \cos\theta \, d\theta = 2dx$

$$\int \frac{2dx}{\sqrt{9-4(x-1)^2}} = \int \frac{3\cos\theta \, d\theta}{3\cos\theta}$$

$$= \int d\theta = \theta + C$$

$$= \boxed{\arcsin\left(\frac{2}{3}(x-1)\right) + C}$$

5. Use the method of partial fractions to find $\int \frac{3+x^2}{x(x^2+1)} dx$.

$$\frac{3+x^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$3+x^2 = A(x^2+1) + (Bx+C)x = Ax^2 + A + Bx^2 + Cx$$

$$1 = A + B$$
$$0 = C$$
$$3 = A \quad \Rightarrow \quad B = -2$$

$$\int \frac{3}{x} dx + \int \frac{-2x}{x^2+1} dx = 3\ln|x| - \ln|x^2+1| + k$$

$$= \boxed{\ln\left(\frac{|x|^3}{x^2+1}\right) + k}$$

6. Does the improper integral $\int_{-1}^{2} x^{-5}\, dx$ converge or diverge? Demonstrate why using limits.

$$\int_0^2 x^{-5} dx = \lim_{t \to 0^+} \int_t^2 x^{-5} dx$$

$$= \lim_{t \to 0^+} -\frac{1}{4} x^{-4} \Big|_t^2$$

$$= \lim_{t \to 0^+} \left[-\frac{1}{4}(2)^{-4} - -\frac{1}{4} t^{-4} \right]$$

$$= \lim_{t \to 0^+} \left[-\frac{1}{64} + \frac{1}{4t^4} \right] = \infty$$

Therefore, $\boxed{\int_{-1}^{2} x^{-5} dx \text{ diverges.}}$

7. Evaluate the improper integral $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+t^2}\, dt.$

$$= \lim_{u\to-\infty} \int_u^0 \frac{1}{1+t^2}\,dt + \lim_{u\to\infty} \int_0^u \frac{1}{1+t^2}\,dt$$

$$= \lim_{u\to-\infty} \arctan t \Big|_u^0 + \lim_{u\to\infty} \arctan t \Big|_0^u$$

$$= \lim_{u\to-\infty} \left[0 - \arctan(u)\right] + \lim_{u\to\infty} \left[\arctan(u) - 0\right]$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \boxed{\pi}$$

8. Use the trapezoid, midpoint, or Simpson's rule (pick your favorite) to estimate

$$\int_{-6}^{6} x^2\, dx\,,$$

with 4 subdivisions.

$$\text{TRAP}(4) = \frac{\Delta x}{2}\left[f(-6) + 2f(-3) + 2f(0) + 2f(3) + f(6)\right]$$

$$= \frac{3}{2}\left[36 + 2\cdot 9 + 2\cdot 0 + 2\cdot 9 + 36\right]$$

$$= \frac{3}{2}\, 108$$

$$= \boxed{162}$$