**MAT123 Practice for Midterm 2**

**Comparison test:**
**1.** Determine whether the following integrals converge.

$$\int_0^\infty \frac{|\sin(x)|}{x^2+1}\,dx \qquad \int_2^\infty \frac{\sqrt{x}}{x-1}\,dx$$

$$\int_0^\infty \frac{e^x}{e^{2x}+x}\,dx$$

**Area:**
**2.** Find the area bounded between the curves $y = \frac{2}{x}$ and $y = 3 - x$.

**3.** Find the area bounded between the curves $x = y^2$ and $y = 1 - x$.

**Volume:**
**4.** Set up but do not evaluate an integral which gives the volume of the donut obtained by revolving a circle of radius 2cm around an axis 4cm from its center.

**5.** Find the volume of a pyramid with height 4m and whose base is a square having sides of length 6m.

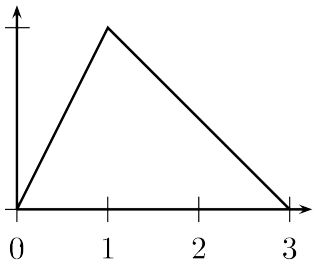
**Work:**
**6.** A chain weighs 4 lb/ft. Calculate the work required to lift one end of the chain to a height of 10 feet.

**Force/Pressure:**
**7.** A fish tank has dimensions 1ft x 2ft x 3ft.
(a) Compute the pressure at depth *h* in the tank.
(b) Compute the force exerted by water on the bottom of the tank.
(c) Compute the force exerted by water on the largest side of the tank.

**Center of mass:**
**8.** A 3m rod has density (kg/m) of $x^2 - 2x + 2$ where $x$ denotes the distance from one end of the rod.
(a) Compute the total mass of the rod.
(b) Compute the center of mass of the rod.

**Statistics:**

**9.** A probability density function is graphed below:




**(a)** Use what you know about the total area under a probability density to find the maximum height of the curve.

**(b)** Find the probability that $1 < x < 3$.

**(c)** Compute the mean.

**(d)** Compute the median.

**Sequences:**

**10.** Write the first 12 terms of the sequence given recursively by

$$a_0 = a_1 = 1 \text{ and } a_{n+2} = (-1)^n a_n + a_{n+1}$$

**11.** Find the limit of the sequence of terms $a_n = \frac{2n}{n+3}$ as $n \to \infty$.

**Series:**

**12.** Describe why the following series converge or diverge:

**(a)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n \cdot (n+1)}{n!}$

**(b)** $\displaystyle\sum_{n=0}^{\infty} \frac{5n^8 - 4n^3 + n - 9}{6n^8 - 3n^2 + 7n + 1}$

**(c)** $\displaystyle\sum_{n=0}^{\infty} \frac{3^n - 1}{n \cdot 5^n}$

**(d)** $\displaystyle\sum_{n=2}^{\infty} \frac{7}{(n^6 - 7)^{1/3}}$

**13.** You stack squares, one on top of the other. Suppose the $n^{th}$ square has sides of length 1/n.

(a) Is there a limit to the height of the stack?

(b) Is there a limit to the total area of the stack?

**14.** Find the limit of the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \cdot 2^n}{3^{n-1}}$.